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COSMOLOGICAL CONSTRAINTS ON HEAVY WEAKLY INTERACTING FERMIONS

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The masses and lifetimes of very heavy weakly interacting fermions which appear in many grand unified gauge models are constrained by the requirement that their decays in the hot big bang early universe should not generate excessive entropy which would dilute n_B/n_γ below its observed value.

Experimental lower limits on the lifetime of the proton indicate that the bosons which should mediate its decay in grand unified gauge models have masses $\gtrsim 10^{14}$ GeV [1]. In this paper we derive cosmological constraints on the properties of heavy fermions ($\mathcal{N}$) which appear in many such models [2, 3]. The presence of baryon-number violating bosons implies that any net baryon number introduced as an initial condition in the standard hot big bang early universe should have been destroyed (e.g., [4]). A baryon asymmetry $0 \leq |n_B/n_\gamma| \leq 1$ may subsequently be generated by *CP*- and *B*-violating decays of heavy bosons (or fermions). In order for a sufficient asymmetry to survive to the present, it is necessary that the number of photons generated at later times not be excessive. Any deviations from thermal equilibrium, which would generate entropy, and hence increase $\rho_\gamma/T_\gamma \sim s \sim n_\gamma$, must be small. However, very weakly interacting particles will not remain in thermal equilibrium when the universe cools. If they survived for long enough between decoupling from equilibrium and decaying, then the entropy released by their decays would dilute n_B/s to below its observed value $\simeq 10^{-9\pm1}$ (even if $n_B/s = O(1)$ before decay). (The expansion of the universe is by assumption adiabatic; its entropy nevertheless increases unless deviations from equilibrium are relaxed away sufficiently quickly for the expansion to be "quasistatic.") To avoid this phenomenon heavy fermions in grand unified theories must have masses and lifetimes which lie outside the shaded region delineated in fig. 1.

According to the standard hot big bang model, any $\mathcal{N}$ should be in thermal equilibrium in the sufficiently early universe, with a number density about that of

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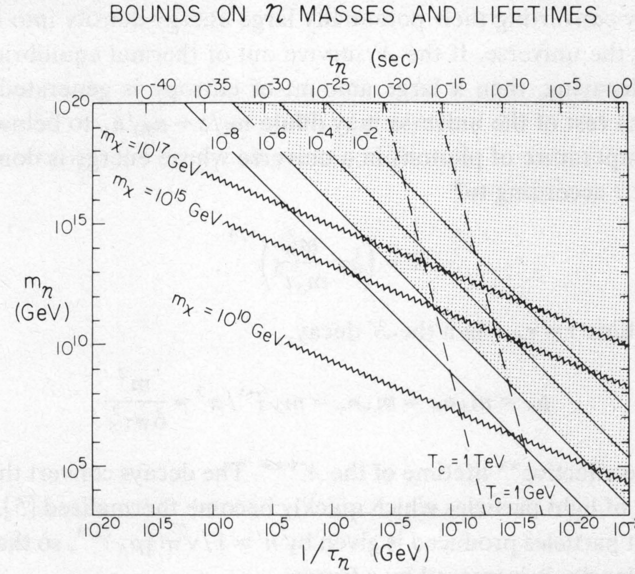

Fig. 1. Masses and lifetimes of weakly interacting particles $\mathcal{N}$ lying within the shaded region are inconsistent with the standard cosmological model. The decays of such $\mathcal{N}$ would generate a large entropy and result in excessive dilution of n_B/n_γ. The solid contour lines give the n_B/s before $\mathcal{N}$ decay necessary to avoid dilution below the observed $n_B/s \gtrsim 10^{-10}$. The wavy lines give lifetimes for $\mathcal{N}$ decaying through coupling to a boson of mass m_χ. The dashed lines give the effective lifetime for $\mathcal{N}$ which decay at a fixed transition temperature T_c.

photons* $n_\gamma \simeq T^3/\pi^2$. If the $\mathcal{N}$ interact with the thermal bath of photons through two-body scattering mediated by the exchange of a particle of mass m_χ and coupling g, they are maintained in equilibrium until $T \lesssim T_D \approx (m_\chi^4/(g^4 m_{\mathcal{P}}))^{1/3}$, when their interaction rate $n_\gamma \sigma \sim g^4 T^5/m_\chi^4$ falls below the expansion rate of the universe T^2/m_P (where the effective Planck mass $m_P = m_{\mathcal{P}}\sqrt{\pi/8\xi}$; $m_{\mathcal{P}}$ is the Planck mass $\simeq 10^{19}$ GeV, ξ is the number of particle species with $m \gg T$ which is typically ~ 100 for $10^2 \lesssim T \lesssim 10^{15}$ GeV in grand unified models). If the $\mathcal{N}$ remain in thermal equilibrium, then their number density would eventually become $\simeq (m_{\mathcal{N}} T)^{3/2} \exp(-m_{\mathcal{N}}/T)$. If $T_D \gtrsim m_{\mathcal{N}}$, the $\mathcal{N}$ decouple from equilibrium while they are still relativistic** with a number density $n_{\mathcal{N}} \simeq n_\gamma \sim T^3$. After decoupling the $\mathcal{N}$ behave as a collisionless gas and their number density remains $\sim n_\gamma$, falling $\sim 1/R^3 \sim T^3$ as the universe expands. The expansion redshifts all momenta $\sim 1/R \sim T$, so that the energy density of photons $\rho_\gamma \sim T n_\gamma \sim T^4$. However, the $\mathcal{N}$ energy density is dominated by the rest mass, so that $\rho_{\mathcal{N}} \sim n_{\mathcal{N}} m_{\mathcal{N}} \sim n_\gamma m_{\mathcal{N}} \sim m_{\mathcal{N}} T^3$; for $\xi T \lesssim m_{\mathcal{N}}$ the $\mathcal{N}$ may dominate the energy density of the universe. Eventually the $\mathcal{N}$

* We assume throughout that all particles obey Maxwell–Boltzmann statistics, and have a single spin state. Ultrarelativistic boson (fermion) spin states would have number densities $\frac{3}{4}(\zeta(3))T^3/\pi^2$; we shall ignore the irrelevant corrections.

** We assume for now that the $\mathcal{N}$ lifetime is much longer than the decoupling time.

decay, thereby converting their potentially large energy density into light particles and reheating the universe. If the $\mathcal{N}$ survive out of thermal equilibrium for a long time before decaying, then a large amount of entropy is generated which when shared with the rest of the universe may dilute $n_B/s \sim n_B/n_\gamma$ to below its observed value. The temperature of photons in a universe whose energy is dominated by $\rho_\mathcal{N}$ falls with time t according to*

$$T \simeq \left(\tfrac{1}{6}\pi \frac{m_\mathcal{P}^2}{m_\mathcal{N} t^2}\right)^{1/3}, \tag{1}$$

so that at the time $t = \tau_\mathcal{N}$ when the $\mathcal{N}$ decay,

$$\rho_\mathcal{N} = m_\mathcal{N} n_\mathcal{N} \simeq m_\mathcal{N} n_\gamma \simeq m_\mathcal{N} T^3/\pi^2 \simeq \frac{m^2}{6\pi\tau_\mathcal{N}^2}, \tag{2}$$

where $\tau_\mathcal{N}$ is the effective** lifetime of the $\mathcal{N}$***. The decays convert this rest energy into a cascade of light particles which quickly become thermalized [5]. The number density of light particles produced is given by $n' \simeq 1/\sqrt{\pi}(\tfrac{1}{3}\rho_\mathcal{N})^{3/4}$, so that the original light particle density is increased by a factor

$$1+\frac{n'}{\xi n_\gamma} \simeq 1+\left(\frac{m_\mathcal{P}^2}{18\pi^{5/3}\tau_\mathcal{N}^2}\right)^{3/4}/\xi n_\gamma$$

$$\simeq \left(\frac{2\tau_\mathcal{N}^2}{9\pi m_\mathcal{P}^2}\right)^{1/4}\frac{m_\mathcal{N}}{\xi}. \tag{3}$$

Any n_B/s present prior to the $\mathcal{N}$ decays (or even produced in the $\mathcal{N}$ decay) would be diluted by this factor through the entropy produced in the decays. The solid contour lines in fig. 1 give the n_B/s prior to the $\mathcal{N}$ decays which would be required in order for the final n_B/s to be $\geqslant 10^{-10}$ as observed at present. Detailed investigations [6] suggest that grand unified models could give a maximum n_B/s before $\mathcal{N}$ decay of about 10^{-6}.

A heavy fermion $\mathcal{N}$ which decays only through a virtual boson χ should have a width given, in analogy to μ decay, by

$$\Gamma_\mathcal{N} = 1/\tau_\mathcal{N} \simeq \left(\frac{g^2}{4\pi}\right)^2 \frac{m_\mathcal{N}^5 |U|^2}{384\pi m_\chi^4}\left(1+\frac{3m_\mathcal{N}^2}{10m_\chi^2}+\dots\right), \tag{4}$$

where $|U|^2 = \mathrm{O}(1)$ represents the sum over "virtual χ decay modes" weighted by requisite mixing angles (including a factor for the number of χ states), and typically

* In general, the scale factor R of a Friedmann universe varies according to $\dot{R}/R = -\dot{T}/T = (8\pi\rho/3m_\mathcal{P}^2)^{1/2}$. If $\rho = \rho_\mathcal{N} + \xi\rho_\gamma = (m_\mathcal{N} + 3T\xi)T^3/\pi^2$, then $t^2 = (\tfrac{1}{6}\pi m_\mathcal{P}^2/m_\mathcal{N}T^3)[(1-2x)\sqrt{1+x}+2x\sqrt{x}]^2$, where $x = \xi T/m_\mathcal{N}$. (For $T \gg m_\mathcal{N}$, this becomes $T \simeq (\tfrac{3}{2}\pi\xi)^{1/4}(m_\mathcal{P}/t)^{1/2}$.) The deviation of this result from eq. (2) is negligible for the large $m_\mathcal{N}$ considered here.

** As discussed below, the temperature of the ambient gas may affect the decay rate of $\mathcal{N}$.

*** In the relevant cases, the $\mathcal{N}$ will be approximately at rest for most of their lifetime, so that time dilation effects are negligible.

$g^2/4\pi \approx 0.02$. This $\tau_{\mathcal{N}}$ is shown by the wavy lines in fig. 1 for various m_χ. If eq. (4) obtains, the light particle density is increased through $\mathcal{N}$ decays by a factor [cf. eq. (3)]

$$1+\frac{n'}{\xi n_\gamma} \simeq \frac{4m_\chi^2}{m_{\mathcal{N}}^{3/2}\, m_{\mathcal{P}}^{1/2}}\,. \qquad (6)$$

For example, if this dilution is to be less than a factor of 10^4 (allowing an initial $n_B/s = 10^{-6}$) then

$$m_{\mathcal{N}} \gtrsim 5\times 10^{-3}\, m_\chi (m_\chi/m_{\mathcal{P}})^{1/3}\,, \qquad (7)$$

so that for $m_\chi = 10^{15}$ GeV, $m_{\mathcal{N}} \gtrsim 2\times 10^{11}$ GeV.

Grand unified models based on extensions of SU(5) (e.g., SO(10), E(6)) almost inevitably involve heavy fermions associated with SU(5) singlet (neutral) components in the fermion representations. In some cases, the $\mathcal{N}$ may be introduced as a heavy right-handed partner for the light ν_L; in other models, the $\mathcal{N}$ may be independent. The former possibility is realized in a class of SO(10) models which provide a natural explanation of small light neutrino masses $m_\nu \approx m_q^2/m_{\mathcal{N}}$ [7] (where m_q is the mass of the relevant charge $\frac{2}{3}$ quark). In these models the dominant decay modes of the $\mathcal{N}$ are $\mathcal{N} \to \varphi^0\nu$, $\bar{\varphi}^0\bar{\nu}$, φ^+e^-, φ^-e^+ (where (φ^+, φ^0) is the usual $SU(2)_L \times U(1)$ Higgs doublet*) yielding a decay width

$$\Gamma_{\mathcal{N}} = 1/\tau_{\mathcal{N}} \sim \alpha (m_q/m_W)^2 m_{\mathcal{N}}\,, \qquad (8)$$

which completely swamps eq. (4), and prevents useful bounds on $m_{\mathcal{N}}$. Limits on the light neutrino masses nevertheless provide some bound on $m_{\mathcal{N}}$. In other models the $\mathcal{N}$ need not be associated with ν_L, although it may still decay to a light $SU(2)_L$ doublet φ (e.g., [9]), thereby avoiding our bounds.

In models with dynamical symmetry breakdown and without explicit Higgs fields, another decay mechanism may be important. The $\mathcal{N}$ may mix with light (virtual) fermions which then "decay" by emission of a light W boson. Typically, such mixing occurs only when the universe has cooled below the critical temperature at which the symmetry which forbids the mixing (through mass terms) is spontaneously broken. The lifetime of the $\mathcal{N}$ in the hot early universe is shown by the dashed line in fig. 1 if the decay proceeds through mixing to light fermions at the indicated transition temperature T_c.

In other possible schemes, the $\mathcal{N}$ cannot decay by mixing, but only by emission of (usually B-violating) bosons. The lifetimes of such $\mathcal{N}$ should follow the wavy lines in fig. 1. If the relevant bosons are more massive than the SU(5) gauge boson ($m \gtrsim 10^{15}$ GeV), our bound indicates that in such schemes $m_{\mathcal{N}} \gtrsim 10^{11}$ GeV.

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* The decay of an $\mathcal{N}$ could produce an average asymmetry of one unit of lepton number (but not baryon number (c.f., [8])).

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