The Decoupling of Axial Mesons From Currents.

S. WOLFRAM

St. John's College - Oxford

(ricevuto il 15 Maggio 1977)

I shall demonstrate that conventional quark models require axial vector mesons to decouple from local currents. It is only possible to avoid this conclusion if the quarks behave in a highly relativistic manner. The main consequences of this result are that heavy-lepton decays and diffractive ν-production are not necessarily good testing grounds for the existence of either the $A1$ or second-class weak currents, and that it may not be reasonable to consider the axial-nucleon form factor to be meson pole dominated.

The momentum-space wave function for an M-meson bound state of an $\bar{\alpha}$ and a β-type quark may be written

$$\chi^{\mathrm{M}}_{\mu\nu}(s, q) = \langle 0|\bar{\psi}^{\alpha}_{\mu}(s+q)\,\psi^{\beta}_{\nu}(s-q)|M\rangle\,, \tag{1}$$

where $2s$ is the M 4-momentum and $2q$ is the $\mathrm{q}\bar{\mathrm{q}}$ relative 4-momentum. The ψ^{γ}_{λ} are momentum-space quark field operators, γ referring to the quark flavour and λ to the spinor component. The wavefunction $\chi^{\mathrm{M}}_{\mu\nu}(s, q)$ has sixteen components, which may be partitioned into 2×2 matrices (¹):

$$\chi^{\mathrm{M}}(\boldsymbol{s}=0,\, s_0 = m_{\mathrm{M}},\, q) = \begin{pmatrix} \chi_{+-} & \chi_{++} \\ \chi_{--} & \chi_{-+} \end{pmatrix}, \tag{2}$$

where the suffices indicate whether the upper (+) or lower (—) components of the quark and antiquark spinors contribute. In the nonrelativistic weak-binding limit (with the representation of the γ matrices used, for example, in BJORKEN and DRELL (²)),

$$\chi_{+-} \sim \chi_{-+} \sim \frac{\boldsymbol{\sigma}\cdot\boldsymbol{q}}{m_{\mathrm{q}}}\,\chi_{++}\,, \qquad \chi_{--} \sim \frac{\boldsymbol{q}^2}{m^2_{\mathrm{q}}}\,\chi_{++}\,. \tag{3}$$

(¹) C. H. LLEWELLYN SMITH: *Ann. of Phys.*, **53**, 521 (1969).

(²) J. D. BJORKEN and S. D. DRELL: *Relativistic Quantum Mechanics* (New York, N. Y., 1964).

The amplitude for the process

(4) 

where J is the local current $\bar{\psi}^\alpha \boldsymbol{I} \psi^\beta$ is given by

$$\langle 0|J(2s)|M\rangle = \mathrm{Tr}\int \chi^M(s, q)\boldsymbol{I}\, \mathrm{d}^4 q\,. \tag{5}$$

The amplitude is thus proportional to the value of the wave function matrix at the original of configuration space $r = 0$. For the electromagnetic and weak vector currents (¹),

$$\langle 0|J^V_\mu(0)|M_\mu(J^{PC} = 1^{--})\rangle \propto \mathrm{Tr}\int \sigma_i(\chi_{++} - \chi_{--})_i\, \mathrm{d}^4 q \tag{6a}$$

and for the axial weak current

$$\langle 0|J^A_\mu(0)|M_\mu(J^{PC} = 1^{+\pm})\rangle \propto \mathrm{Tr}\int \sigma_i(\chi_{+-} - \chi_{-+})_i\, \mathrm{d}^4 q \tag{6b}$$

$$\langle 0|J^A_\mu(0)|M(J^{PC} = 0^{-+})\rangle \propto \mathrm{Tr}\int (\chi_{++} - \chi_{--})_i\, \mathrm{d}^4 q\,. \tag{6c}$$

Hence in the nonrelativistic limit for the χ_{ab} (in which these can only be significant around $q = 0$), axial mesons must decouple from the W (and Z). Mesons with $J^{PC} = 1^{+-}$ are « second-class », so that they may only couple to second-class currents of the form $\boldsymbol{I} = 2s_\nu \sigma^{\mu\nu}\gamma_5$. These give

$$\langle 0|J^2_\mu(0)|M_\mu(J^{PC} = 1^{+-})_- \propto \mathrm{Tr}\int \sigma_i(\chi_{+-} + \chi_{-+})_i\, \mathrm{d}^4 q \tag{6d}$$

which again suggests decoupling.

The matrix χ may in principle be calculated using the Salpeter-Bethe (SB) equation, but in fact this is not possible in any general physically reasonable case. The Wick-rotated SB equation with an harmonic oscillator kernel and the approximation $\langle \boldsymbol{q}^2\rangle/m_q^2 = 0$ has been solved by Bohm, Joos and Krammer (³). Not surprisingly, they obtain exact decoupling. Another typical solvable model of mesons is the rigid-walled bag of Preparata and Craigie (⁴), which again predicts exact decoupling. The conventional assumption that $\langle \boldsymbol{q}^2\rangle/m_q^2 \ll 1$ is supported by the fact that spin-orbit splittings between $q\bar{q}$ states, which are proportional to $\langle \boldsymbol{q}^2\rangle/m_q^2$, tend to be small, and also by the apparent success of nonrelativistic methods in the cc system (although comparable success is not possible with lighter quarks).

The direct experimental consequences of the result that (for conventional $q\bar{q}$ potentials) $J^{PC} = 1^{+\pm}$ mesons decouple from local currents are that these mesons cannot be produced either in the decay $L \to M\nu_L$ (L is a heavy lepton) or diffractively in reactions like $\nu\mathcal{N} \to {}^{\mu}_{\nu}\, \mathcal{N}M$. If the A1 really exists (*), then typical models predict

(³) M. Bohm, H. Joos and M. Krammer: *Nucl. Phys.*, **51** B, 397 (1973).

(⁴) G. Preparata and N. S. Craigie: CERN preprint, CERN-TH-2038 (1975).

(*) The Probable discovery of a 1^{++} $c\bar{c}$ state renders its possible nonexistence yet more mysterious.

$B(L \to Al\nu_L) \sim B(L \to \pi\nu_L) \sim 10\%$ (for $m_L = 2$ GeV), so long as the $L \to \nu_L$ current contains the usual axial part. I predict that the former decay should be at least strongly suppressed. Note, however, that my result does not forbid nonresonant $\pi^+\pi^+\pi^-$ production in L decay. The decay $L \to B\nu_L$ has been mentioned as a test for second-class currents; it should not in fact occur even in their presence. Diffractive neutrino-production of the Al (and F_A^*) has been discussed by many authors ([5], and references therein). In this process, $J \to Al$ (K_A^*, F_A^*, χ_A ...) occurs slightly off the meson mass shell, but the rapid fall-off of hadronic diffraction with t ensures that it happens not far from the mass shell. Hence my result applies, and there should be almost no axial meson production, as opposed to a rate comparable with vector meson production implied by the usual model. Diffractive B neutrino-production, which has been suggested [6] as a test for second-class currents, should also not occur. Again, nonresonant production of the decay products of axial mesons is not affected by my results, and in fact, if one assumes that my result applies to the Al, then the appearance of an enhancement in $m^2_{\pi\pi\pi}$ in $\nu\mathcal{N} \to \mu\mathcal{N}\pi^+\pi^+\pi^-$ or $L \to \nu_L\pi^+\pi^+\pi^-$ would therefore suggest that the Al is *not* a genuine particle.

My result also shows that, even if the Al and its partners exist, it should not be a good approximation to saturate the axial current with these meson poles, so that axial-meson dominance must be incorrect. However, it is believed that the axial-nucleon form factor is qualitatively similar to the vector ones, suggesting that in none of these cases should meson dominance be used. Note that the longitudinal (derivative) contribution to the axial form factor is $O(m_q^2/q^2)$ and thus insignificant.

* * *

I am grateful to C. H. LLEWELLYN SMITH for discussions and for reading the manuscript; also to F. E. CLOSE, P. S. FAGE and D. M. SCOTT for comments.

---

(5) M. K. GAILLARD, S. A. JACKSON and D. V. NANOPOULOS: *Nucl. Phys.*, **102** B, 326 (1976); M.-S. CHEN, F. S. HENYEY and G. L. KANE: *Nucl. Phys.*, **118** B, 345 (1977).
(6) M.-S. CHEN, F. S. HENYEY and G. L. KANE: *Nucl. Phys.*, **114** B, 147 (1976).

S. Wolfram
3 Settembre **1977**
Lettere al Nuovo Cimento
Serie 2, Vol. **20**, pag. 10-12