INTRODUCTION TO THE WEAK INTERACTION

VOLUME TWO

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C O P Y   T W O

V O L U M E   T W O

(Chapters 6 - 9)

CHAPTER SIX: WEAK INTERACTIONS OF THE HADRONS.

6.1 Pion Decay.

In 1947 Lattes, Muirhead, Ochialini and Powell (1) obtained tracks of charged pions in nuclear emulsions. These appeared to decay into muons after they had travelled a short distance in the emulsion. By measuring the multiple scattering in the tracks due to Coulomb repulsion by emulsion nuclei, the pion mass was estimated to be about 140 MeV/c^2, in accordance with the predictions of Yukawa (2). The first accurate measurement of the charged pion mass was made by magnetic analysis[1] of a secondary pion beam from an accelerator, and this yielded a mass of 139.6 MeV/c^2. The currently acknowledged value for the charged pion mass is (3)

$$139.5688 \pm 0.0064 \quad MeV/c^2 \ . \tag{6.1.1}$$

The charged pion lifetime has been measured as

$$(2.6030 \pm 0.0023) \times 10^{-8} \ s \ . \tag{6.1.2}$$

We now attempt to construct a Hamiltonian for the common charged pion decay

$$\pi^{\pm} \longrightarrow \mu^{\pm} + \bar{\nu} \ . \tag{6.1.3}$$

Obviously this could be done by writing a product of the three relevant fields and then multiplying by a suitable coupling constant. However, there is no need to introduce a new interaction. We know that the pion interacts strongly with nucleons. Thus we postulate that the charged pion decay is a two-stage process: first the pion virtually decays into nucleon-antinucleon pair (it cannot actually decay into this channel because of mass conservation), and then the nucleon-antinucleon pair annihilate each other by a reordered neutron decay reaction. The quantity obtainable from a Hamiltonian and most readily comparable with experiment is usually the lifetime or matrix element for a decay. In the case of the charged pion, however, it is difficult to obtain a complete matrix element for decay because very little is known about the

$$\pi \longrightarrow \bar{N} + N \tag{6.1.4}$$

strong interaction vertex. However, we shall now consider the matrix element for the second vertex, which is purely due to the weak interaction. We write the Hamiltonian in the usual four-fermion form:

$$H_I = (g/\sqrt{2}) \quad \bar{\psi}_N(x)\gamma_r(1 + \gamma_5)\psi_N(x)\,\bar{\psi}_\nu(x)(1 - \gamma_5)\gamma_r\,\psi_\mu(x) + \\ + \text{Herm. conj.} \qquad (6.1.5)$$

The Hamiltonian (6.1.5) allows us to make a prediction immediately. Since we know that the antineutrino in π^+ decay will be right-handed, and as we also know that, in this decay, the muon and antineutrino must emerge in opposite directions with zero total angular momentum, we may deduce that the muon will have negative helicity. The situation is reversed for the π^-. Our helicity theory has been checked experimentally (4) and has been found to be correct. The existence of definite polarization for the particles in pion decay shows that here also, parity is not conserved. This hypothesis was tested by Garwin et al. (5) in 1957. A beam of pions was allowed to decay into muons. According to (6.1.5), these will tend to be aligned in a particular direction. When the muons decayed, the counting rate for electrons was measured from all directions, and it was found that the electrons were preferentially emitted in one direction, demonstrating parity violation.

We write the complete matrix element for pion decay as

$$M_{if} = \sum_{N,\bar{N}} \langle \mu, \bar{\nu}_\mu \mid H_I \mid N, \bar{N} \rangle \langle \bar{N}, N \mid X \mid \pi \rangle \ , \qquad (6.1.6)$$

where X is the unknown strong interaction Hamiltonian at the first vertex. From (6.1.5), we see that the first matrix element on the right-hand side of (6.1.6) is given by

$$\langle \mu, \bar{\nu}_\mu \mid H_I \mid N, \bar{N} \rangle = (g/\sqrt{2})\ \bar{u}_N^{(-)}(-\underline{p}')\gamma_r(1 + \gamma_5)\,u_N^{(+)}(\underline{p}) \ \times \\ \times \ \bar{u}_\mu^{(+)}(\underline{p}_\mu)(1 - \gamma_5)\gamma_r\,u_\nu^{(-)}(-\underline{p}_\nu)\ . \int d^3x \exp\left(j(p + p' - p_\mu - p_\nu)x\right) \qquad (6.1.7)$$

Using reduction formula techniques, we may tentatively write the matrix element for X as

$$\langle \bar{N}, N \mid X \mid \pi \rangle = (2\pi)^4\ \delta(p + p' - p_\pi)\,\bar{u}_N^{(+)}(\underline{p})\,F(p, p', p_\pi)u_N^{(-)} \ \times \\ \times (-\underline{p}')\,(1/(\sqrt{2E_\pi})) \ , \qquad (6.1.8)$$

where E_π is the energy of the pion and $F(p, p', p_\pi)$ is an unknown function of the nucleon and pion momenta. Putting (6.1.7) and (6.1.8) together, substituting in (6.1.6), summing over the nucleon quantum numbers, and finding F using Lorentz invariance, we finally obtain the transition rate for pion decay when the muon is emitted with polarization r in the solid angle $d\Omega$:

$$\frac{\delta W}{\delta t} = \frac{g^2 f^2}{4\pi} \frac{d\Omega}{4\pi} \frac{m_{\mu}^2}{m_{\pi}^2} E_{\mu} |p_{\mu}| \bar{u}_{\mu}^{(+)(r)}(p_{\mu})(1 + \gamma_5) \times (\gamma_4 m_{\pi} - m_{\mu}) u_{\mu}^{(+)(r)}(p_{\mu}) , \qquad (6.1.9)$$

where

$$|p_{\mu}| = m_{\pi} - E_{\mu}, \qquad (6.1.10)$$

and where g is the weak coupling constant and f is a number associated with the strong interaction. Integrating over all angles and summing over the possible spin directions for the outgoing muon, (6.1.9) becomes

$$\frac{1}{T_{\pi}} = \frac{g^2 f^2}{2\pi} \frac{m_{\mu}^2}{m_{\pi}^2} E_{\mu} |p_{\mu}| (m_{\pi} - (m_{\mu}^2 / E_{\mu}))$$
$$= \frac{g^2 f^2}{8\pi} \frac{m_{\mu}^2}{m_{\pi}^3} (m_{\pi}^2 - m_{\mu}^2)^2 . \qquad (6.1.11)$$

However, (6.1.11) does not allow us to predict a value for T_{π}, since we do not know the value of f. Substituting for T_{π}, we obtain

$$f = 0.931\, m_{\pi} , \qquad (6.1.12)$$

which is confirmed by a study of the strong interaction.

6.2 Electron-Muon Universality in Pion Decay.

The principle of electron-muon universality states that all weak couplings are identical, and hence that, to within phase-space and kinematical factors, the electron and muon should be interchangeable in any reaction, and the interchange should not alter the matrix element for that reaction. Consequently, a decay mode of the type

$$\pi^{\pm} \longrightarrow e^{\pm} \ v \qquad (6.2.1)$$

was searched for, and was found to have a branching ratio of

$$(1.24 \pm 0.03) \times 10^{-4} \ \% . \qquad (6.2.2)$$

Recalling (6.1.11), and substituting m_e for m_{μ}, we obtain

$$\frac{1}{T_{\pi}'} = \frac{g^2 f^2}{8\pi} \frac{m_e^2}{m_{\pi}^3} (m_{\pi}^2 - m_e^2)^2 , \qquad (6.2.3)$$

so that we predict the branching ratio

$$\frac{R(\pi \longrightarrow \mu + v)}{R(\pi \longrightarrow e + v)} \qquad (6.2.4)$$

as

$$T/T' = (m_e/m_\mu)^2 \ \left((m_\pi^2 - m_e^2)/(m_\pi^2 - m_\mu^2) \right)^2 \approx 1.283 \times 10^{-4} , \quad (6.2.5)$$

in excellent agreement with the experimental value (6.2.2).

At this juncture, we consider the sensitivity of (6.2.5) under the inclusion of contributions other than the usual vector and axial vector terms. From (6.1.6), (6.1.7) and (6.1.8), we obtain, removing the assumption of pure V - A interaction:

$$\langle \mu, \bar{v}_\mu | S | \pi \rangle = (2\pi)^4 \ \delta(p_\mu + p_v - p_\pi) \ (1/(2\sqrt{E_\pi}) \sum_i g_i \ F_i(p_\pi) \quad \mathrm{X}$$
$$\mathrm{X} \quad \bar{u}_\mu^{(+)}(\underline{p}_\mu) \ O_i \ (1 + a_i \gamma_5) \ u_v^{(-)}(-\underline{p}_v) \quad , \quad (6.2.6)$$

where g_i are the coupling constants for parity-conserving terms and a_i are the coupling constants for parity-violating ones. F_i is a function obtained from F in (6.1.8). From symmetry considerations, we find that only the axial vector and pseudoscalar terms F_i will make nonvanishing contributions. Thus we may rewrite (6.2.6) as

$$\langle \mu, \bar{v}_\mu | S | \pi \rangle = (2\pi)^4 \ \delta(p_\mu + p_V - p_\pi) \ (g/2\sqrt{E_\pi}) \ \bar{u}_\mu^{(+)}(\underline{p}_\mu) \quad \mathrm{X}$$
$$\mathrm{X} \quad (m_\mu f_V - f_S + \gamma_5(m_\mu a_V f_V - a_S f_S) \ u_v^{(-)}(-\underline{p}_v) \quad (6.2.7)$$

Using the standard formula (3.4.10), integrating over muon and neutrino momenta, and using the properties of the delta function, we obtain

$$1/T = (g^2/16\pi) \ (m_\pi^2 - m_\mu^2)^2/(m_\pi^3) \ (|m_\mu f_V - f_S|^2 + $$
$$+ |m_\mu a_V f_V - a_S f_S|^2 . \quad (6.2.8)$$

We now assume that the neutrino spinor has only two components (see 3.8), so that

$$a_V = a_S = 1 . \quad (6.2.9)$$

The assumption (6.2.9) reduces (6.2.8) to

$$1/T = (g^2/8\pi) \ (m_\pi^2 - m_\mu^2)^2/(m_\pi^3) \quad |m_\mu f_V - f_S|^2 . \quad (6.2.10)$$

We note that (6.2.10) simplifies to (6.1.11) if we assume

$$f_S = 0 \quad . \quad (6.2.11)$$

By electron-muon universality, the result (6.2.10) also applies to the decay (6.2.1) if we replace m_μ by m_e. Thus the branching ratio (6.2.4) now becomes

$$T_\pi/T'_\pi = ((m_\pi^2 - m_\mu^2)/(m_\pi^2 - m_e^2))^2 \quad |(m_e f_V - f_S)/(m_\mu f_V - f_S)|^2 , \tag{6.2.12}$$

which agrees with (6.2.5) in the special case (6.2.11). As soon as f_S begins to make a significant contribution, the branching ratio becomes near to unity, in violent disagreement with the experimental value (6.2.2). Thus we are forced to conclude that the condition

$$f_S = 0 \tag{6.2.13}$$

is fulfilled, so that there are no pseudoscalar terms in the pion decay Hamiltonian or matrix element. Since the vertex (6.1.4) is not necessarily nonrelativistic, any pseudoscalar contributions would appear here, and would cause incorrect results. This furnishes another proof that there exist no pseudoscalar terms in the ordinary beta decay Hamiltonian. There exists one further important pion decay mode:

$$\pi^\pm \longrightarrow \pi^\circ + e^\pm + \nu . \tag{6.2.14}$$

The Hamiltonian for the beta decay (6.2.14) is similar to that for neutron decay (3.3.5). (6.2.14) has a branching ratio of (6)

$$(1.02 \pm 0.07) \times 10^{-8} \quad s . \tag{6.2.15}$$

6.3 The Decay of the Charged Kaon.

Following the discovery of massive particles producing 'V-shaped' tracks by Leprince-Ringuet et al. (7) in 1944, the Bristol cosmic ray group (8) and subsequently O'Cealleigh (9) obtained the tracks of particles with a mass of about 500 MeV/c^2 in nuclear emulsions. The new particles were named kaons. Using momentum-analysing magnets to measure kaon momenta, the kaon mass has been established as (10)

$$493.707 \pm 0.037 \quad MeV/c^2 \tag{6.3.1}$$

for the charged kaons, and

$$497.70 \pm 0.13 \quad MeV/c^2 \tag{6.3.2}$$

for the K^o. The lifetime of the $K^\pm$ has been measured as

$$(1.2371 \pm 0.0026) \times 10^{-8} \quad s \tag{6.3.3}$$

by time-of-flight methods[2]. To date, a considerable number of charged kaon decay modes have been discovered, and we tabulate these, together with their branching ratios:

Decay Mode	Branching Ratio
$\mu \nu$	(63.54 ± 0.19) %
$\pi \pi^0$	(21.12 ± 0.17) %
$\pi \pi^- \pi^+$	(5.59 ± 0.03) %
$\pi \pi^0 \pi^0$	(1.73 ± 0.05) %
$\mu \pi^0 \nu$	(3.20 ± 0.09) %
$e \pi^0 \nu$	(4.82 ± 0.05) %
$e \pi^0 \pi^0 \nu$	$(1.8 \pm 1.5) \times 10^{-5}$
$\pi \pi^{\mp} e^{\pm} \nu$	$(3.7 \pm 0.2) \times 10^{-5}$
$\pi \pi^{\pm} e^{\mp} \nu$	$(< 5) \times 10^{-7}$
$\pi \pi^{\mp} \mu^{\pm} \nu$	$(0.9 \pm 0.5) \times 10^{-5}$
$\pi \pi^{\pm} \mu^{\mp} \nu$	$(< 3) \times 10^{-6}$
$e \nu$	$(1.38 \pm 0.20) \times 10^{-5}$
$e \nu \gamma$	$(< 7) \times 10^{-5}$
$\pi \pi^0 \gamma$	$(2.71 \pm 0.19) \times 10^{-4}$
$\pi \pi^+ \pi^- \gamma$	$(10 \pm 4) \times 10^{-5}$
$\mu \pi^0 \nu \gamma$	$(< 6) \times 10^{-5}$
$e \pi^0 \nu \gamma$	$(3.7 \pm 1.4) \times 10^{-4}$
$\pi e^+ e^-$	$(< 0.26) \times 10^{-6}$
$\pi^{\mp} e^{\pm} e^{\pm}$	$(< 1.5) \times 10^{-5}$
$\pi \mu^+ \mu^-$	$(< 2.4) \times 10^{-6}$
$\pi \gamma \gamma$	$(< 3.5) \times 10^{-5}$
$\pi \gamma \gamma \gamma$	$(< 3) \times 10^{-4}$
$\pi \nu \bar{\nu}$	$(< 0.6) \times 10^{-6}$
$\pi \gamma$	$(< 4) \times 10^{-6}$
$e \pi^{\mp} \mu^{\pm}$	$(< 3) \times 10^{-8}$
$e \pi^{\pm} \mu^{\mp}$	$(< 1.4) \times 10^{-8}$
$\mu \nu \nu \bar{\nu}$	$(< 6) \times 10^{-6}$

We shall consider the decays listed above from a number of different angles. We begin with an ordinary transition rate calculation for the

$$K^{\pm} \longrightarrow \mu^{\pm} + \nu \qquad (6.3.4)$$

decay. We simply rewrite the pion decay transition rate (6.1.11) for the kaon:

$$1/T_K = (g^2 f_K^2)/(2\pi) \quad (m^2_{\mu})/(m_K^3) \quad (m_K^2 - m_{\mu}^2)^2 . \qquad (6.3.5)$$

First, we calculate the ratio

$$(f_K/f_{\pi})^2 , \qquad (6.3.6)$$

and this we find, substituting the value (6.3.3) for the charged kaon lifetime in (6.3.5) to be

$$\sim 1/14 . \qquad (6.3.7)$$

Thus f_K^2 is about an order of magnitude less than f_{π}^2. We may think of the K decay process, like that of the pion, as a two-stage reaction:

$$K^+ \longrightarrow p + \overline{\Lambda} \longrightarrow e^+ + v . \qquad (6.3.8)$$

As we shall see later, the second vertex of this reaction is supressed, which accounts for the low value of f_K. We now consider the decay (6.3.4), and hence (6.3.8), in terms of currents. According to the V - A theory, the current $\overline{\Lambda}p$ contains both vector and axial vector terms in various proportions. However, only one of these types of current is manifest in the decay (6.3.4). If the K^+ parity is odd, then the decay will occur via the axial or pseudovector interaction, and if it is even, through the vector one. The Sakata model (see chapter 7) predicts that the parity of the kaon should be the product of the parities of the nucleon and $\overline{\Lambda}$ and their orbital parity. From a study of helium-4 hyperfragments[3] it has been established (11) that the parity of the charged kaon is -1, and hence the axial vector term is responsible for the decay (6.3.4).

The transition rate for

$$K^{\pm} \longrightarrow e^{\pm} + v \qquad (6.3.9)$$

may also be calculated from (6.1.11). The ratio of the transition rates of (6.3.9) to (6.3.4) is important. Using a modified version of (6.1.11) for the individual transition rates, the ratio becomes

$$W(K_{e2})/W(K_{\mu 2}) = (m_e^2/m_{\mu}^2) \; ((m_K^2 - m_e^2)/(m_K^2 - m_{\mu}^2))^2$$
$$\approx 2.58 \times 10^{-5} . \qquad (6.3.10)$$

The current experimental value for this ratio is (12)

$$(1.95 \pm 0.65) \times 10^{-5} , \qquad (6.3.11)$$

agreeing, within the limits of error, with (6.3.10). The fact that the charged kaon parity is -1 does not rule out the possibility of a pseudoscalar interaction in both (6.3.4) and (6.3.9). However, a pure pseudoscalar interaction would suggest

$$W(K_{e2})/W(K_{\mu 2}) = (m_K^2 - m_e^2)^2/(m_K^2 - m_\mu^2)^2 \sim 1.02 . \qquad (6.3.12)$$

(6.3.12) is in complete disagreement with experimental results, and by comparing this to the measured ratio, we find that the upper limit on the admixture of pseudoscalar hadron current is 2.3×10^{-3}. We note that the muon helicity in $K_{\mu 2}$ decay is found (13) to be identical to that in $\pi_{\mu 2}$ decay, and, furthermore, these decay modes behave similarly with respect to high-energy v_μ scattering. The leptonic kaon decay modes all involve $\Delta Y = 1$, whereas those for the pion are all of the form $\Delta Y = 0$. Thus the similarities mentioned above indicate that the same leptonic couples both with the $\Delta Y = 0$ and with the $\Delta Y = 1$ hadron currents.

The next decay mode which we consider is

$$K^{\pm} \longrightarrow e^{\pm} + \pi^{\circ} + v . \qquad (6.3.13)$$

We now attempt to determine the matrix element for the decay (6.3.13). We write

$$V_i = f_1 \, p_{K_i} + f_2 \, p_{\pi_i} , \qquad (6.3.14)$$

where f_1 and f_2 are two arbitrary functions. We have written V_i because we are concerned with a pure vector interaction, since the parity of the $K^{\pm}$ is the same as that of the π°. Thus

$$A_i = 0 . \qquad (6.3.15)$$

Multiplying the four-vector (6.3.14) by the leptonic current

$$\bar{u}_e \, \gamma_i \, (1 + \gamma_5) \, u_v , \qquad (6.3.16)$$

and using

$$p_\pi = p_K - q , \qquad (6.3.17)$$

$$q_i \, \bar{u}_e \, \gamma_i (1 + \gamma_5) \, u_v = m_e \, \bar{u}_e \, (1 + \gamma_5) \, u_v = 0 , \qquad (6.3.18)$$

assuming zero electronic mass, we obtain the matrix element

$$M_{if} = G \sqrt{2} \; g(q^2) \, \psi_\pi \, \psi_K \, p_{K_i} \, \bar{u}_v \, \gamma_i \, (1 + \gamma_5) \, u_e , \qquad (6.3.19)$$

where g is an unknown form factor. Since

$$q^2 = (p_K - p_\pi)^2 = m_K^2 + m_\pi^2 - 2m_K E_\pi , \qquad (6.3.20)$$

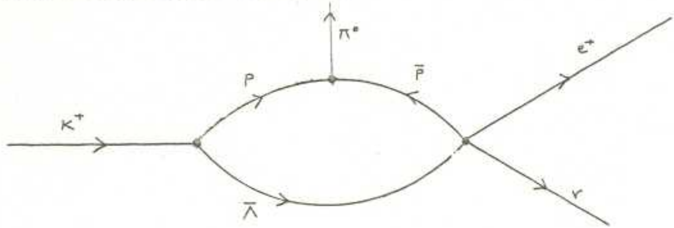
we may write g as a function of E_π instead of q^2. Obviously

$$2g = f_1 + f_2 . \qquad (6.3.21)$$

We now wish to find the transition rate for (6.3.13). Summing over the lepton spin states, integrating over the outgoing particles' momenta, and using the expression for the pion energy spectrum, we obtain:

$$dW = (m_K G^2 g^2 p^3 dE_\pi)/(12\pi^3) \quad , \tag{6.3.22}$$

where p is the momentum of the original $K^\pm$. We assume that $g(E_\pi)$ is a slowly-varying function, at least for the energy range encountered in kaon decay. The presence of the form factor g is associated with the existence of such virtual strong interaction loops as



(6.3.23)

The virtual baryons in the loop of the Feynman diagram (14) (6.3.23) have masses substantially larger than the total energy of the pion. Thus we are justified in assuming that the momentum integral for these particles will not vary over any large amount as the energy of the emitted pion varies. Hence g will effectively be a constant. Integrating (6.3.22) for constant g, we obtain

$$W = (G^2 g^2 m_K^5)/(768\pi^3) \quad . \tag{6.3.24}$$

We now include a 'suppression factor' of 0.6, caused by the nonzero value of the pion mass, so that (6.3.24) reads

$$W = (G^2 g^2 m_K^5)/(768\pi^3) \,.\, 0.6 \quad . \tag{6.3.25}$$

Using the experimental result for W (6.3.3), we find that

$$g^2 \sim 2.5 \times 10^{-2} \quad , \tag{6.3.26}$$

for the decay (6.3.13). However, the corresponding pion decay (6.2.14) gives

$$g = 2 \,, \tag{6.3.27}$$

indicating that $\Delta Y = 1$ decays are suppressed by at least an order of magnitude in comparison with hypercharge-conserving decays. We now consider the evidence for a pure vector interaction in K_{e3} decay. In order to do this, we must find some measurement which is completely independent of $g(E_\pi)$ and hence of any assumptions which we may make concerning its form. Such a measurement is the polarization of the electrons in the decay. We disregard the electron mass, as we did in writing our matrix element (6.3.19). Thus it may be considered as a two-component particle similar to the neutrino. Since

$$v_{e^{\pm}} \sim c \tag{6.3.28}$$

in K_{e3} decay, the electron helicity will effectively be -1. However, there are a number of rare cases in which this is not correct. These occur when the pion carries away no energy, leaving the neutrino and electron to emerge from the decay in opposite direction and with opposite helicities, because of the conservation of angular momentum. If the electron mass were actually zero, then such a situation would evidently be forbidden. However, the slight departure of the electron mass from zero causes this situation occasionally to happen. Experimentally, it is necessary to measure the polarization of positrons in K^+ decay, since nearly all K^- particles will be captured by atomic nuclei. Here the vector current gives right-handed positrons, whereas the tensor and scalar currents, which are the only other possible terms in K_{e3} decay, predict left-handed positrons. Experiments show that the positrons are in fact right-handed, confirming the vector current model.

Another feature of K_{e3} decay which is independent of assumptions about the form factor g is the electron-energy spectrum for a given pion energy. From the matrix element (6.3.19), we may deduce that the electron spectrum for a particular pion energy is given by (15)

$$dW = (G^2 g^2 m_K)/(8\pi^3) \quad (p_\pi{}^2 - (m_K - E_\pi - 2E_e)^2)\, dE_\pi\, dE_e \; . \tag{6.3.29}$$

Thus the energy of an electron for fixed pion energy varies within the range

$$(m_K - E_\pi - |p_\pi|)/2 \leqslant E_e \leqslant (m_K - E_\pi + |p_\pi|)/2 \, . \tag{6.3.30}$$

The electron spectrum becomes zero at both the maximum and minimum points given in (6.3.30), and has a peak at

$$E_e = E_{peak} = (m_K - m_\pi)/2 \; . \tag{6.3.31}$$

For a vector coupling, the energy spectrum is of the approximate form

$$N = -E_e^2 + E_{peak} \; , \tag{6.3.32}$$

whereas, for a scalar coupling it is

$$N = E_{peak} \qquad (E_e \leqslant E_{max}) \, , \tag{6.3.33}$$

and for a tensor coupling it is

$$N = E_e^2 \; . \tag{6.3.34}$$

Thus we see that a measurement of the electron spectrum for a fixed pion energy would enable the coupling responsible for K_{e3} decay to be determined unambiguously.

However, the number of events with a particular pion energy tends to be small, and so we must devise a method of using events with different, but known, pion energies. This problem was considered in detail by Kobzarev (16). The best method was found to utilize a Dalitz plot (17). Here energies are plotted for each of the three resultant particles within an equilateral triangle. The axes are the perpendicular bisectors of the sides. Geometrically, we see that the sum of the distances from a point on to the three sides is always a constant equal to the height of the triangle. This arises from the fact that the lines going to each side of the triangle are the altitudes of three smaller triangles whose total area is equal to that of the large triangle. In three-body decays, this property is useful, since the height of the triangle is interpreted as the energy of the initial particle, and energy conservation is automatically obeyed by any point within the triangle. However, not every point in the triangle will be allowed, because of momentum conservation. We draw an axis x coincident with the base of the triangle, and a y axis with its perpendicular bisector. We let

$$h = m_K - m_\pi \ , \qquad (6.3.35)$$

$$y = T_\pi = E_\pi - m_\pi \ , \qquad (6.3.36)$$

$$x = (E_\nu - E_e)/\sqrt{3} \ . \qquad (6.3.35)$$

The allowed region within the triangle is bounded by the line corresponding to the maximum kinetic energy of the pion,

$$y = (m_K - m_\pi)^2/(2m_K) \ , \qquad (6.3.36)$$

and by that corresponding to the maximum energy of the other two products:

$$3x^2 = y^2 + 2m_\pi y \ . \qquad (6.3.37)$$

We see that for any function g, the total number of electrons with an energy within the range

$$0 \leqslant E_e \leqslant \tfrac{1}{2} E_{max\,(e)} \qquad (6.3.38)$$

must be lower than that with an energy of

$$\tfrac{1}{2} E_{max\,(e)} \leqslant E_e \leqslant E_{max\,(e)} \ , \qquad (6.3.39)$$

since the diagram must be symmetrical about the vertical (y) axis, because the zero-mass electron and neutrino spectra must be identical. We now modify our diagram, defining

$$y' = |p_\pi| \ , \qquad (6.3.40)$$

$$x' = |E_v - E_e| \quad , \tag{6.3.41}$$

we see that the allowed region takes up the slightly simpler form of the area bounded by the y' axis and the lines

$$y' = y'_{max} \quad , \tag{6.3.42}$$

and

$$y' = x \quad . \tag{6.3.43}$$

In terms of our new variables, the energy distribution (6.3.29), assuming the first factor to be roughly unity, simplifies to

$$dW \sim (y'^2 - x'^2)\, dx'\, dy' \quad . \tag{6.3.44}$$

Let us now draw a ray in our modified diagram such that

$$x' = a\, y' \quad , \tag{6.3.45}$$

$$0 \leqslant a \leqslant 1 \quad . \tag{6.3.46}$$

We calculate the ratio of the total number of points to the left of this ray to the total number of points to its right. For a vector coupling,

$$R_V = (3/2)\,(a - (a^3/3)) \quad , \tag{6.3.47}$$

for a scalar coupling

$$R_S = a \quad , \tag{6.3.48}$$

and for a tensor coupling

$$R_T = a^3 \quad . \tag{6.3.49}$$

Experimentally, it has been found (18) that the distribution ratio (6.3.47) is definitely favoured, indicating, once again, a pure vector coupling in K_{e3} decay.

We now consider the decay mode

$$K^{\pm} \longrightarrow \mu^{\pm} + \pi^{\circ} + v \quad . \tag{6.3.50}$$

Since the muon mass may not be disregarded in the same manner as m_e may, an extra term

$$-f\, m_\mu\, \bar{u}_v\, (1 - \gamma_5)\, u_\mu \tag{6.3.51}$$

enters into the matrix element (6.3.19), so that the latter becomes

$$M_{if} = G\sqrt{2}\, \psi_K\, \psi_\pi\, (g\, p_{K_i} - f\, q_i)\, \bar{u}_v\, \gamma_i\, (1 + \gamma_5)\, u_\mu \quad , \tag{6.3.52}$$

where q is here the total momentum of the leptons. If we assume the functions f and g to be effective constants, then we finally obtain

$$W_{\mu 3} \simeq (G^2\, m_K^5)/(768\,\pi^3)\, (0.5g^2 - 0.2fg + 0.05f^2) \quad . \tag{6.3.53}$$

Since we know that, experimentally, the probabilities (6.3.53) and (6.3.25) are approximately equal, we may write

$$0.6\, g^2 \sim 0.5\, g^2 - 0.2\, fg + 0.05\, f^2 \qquad (6.3.54)$$

by equating the two probabilities. The equation (6.3.54) has two solutions for f/g:

$$f/g \sim 4.5 \;, \qquad (6.3.55)$$

$$f/g \sim -0.5 \;. \qquad (6.3.56)$$

For muon kinetic energies of under 30 MeV, the muon spectra corresponding to the different possible values of f/g differ by a factor of nearly 3. At large muon energies, the two spectra tend to become similar. The muon polarization is even more sensitive to the value of f/g. The solution (6.3.55) corresponds to a predominantly scalar interaction, while (6.3.56) corresponds to a vector coupling. If the solution (6.3.56) is correct, then the muons in $K^-_{\mu 3}$ decay should have a predominantly left-handed polarization. If, however, a scalar interaction predominates, then the muons will be mostly right-handed. Thus, by observing muon asymmetry, we may deduce the type of coupling responsible for $K_{\mu 3}$ decay. Experiments (19) favour a vector interaction.

We now consider the $K_{2\pi}$ and $K_{3\pi}$ decays

$$K \longrightarrow \pi + \pi^0 , \qquad (6.3.57)$$

$$K \longrightarrow \pi + \pi + \pi \qquad (6.3.58)$$

in terms of isospin, which we discussed in 5.1. We shall calculate the matrix elements for these decays in 7.1. Since the final state in the decay (6.3.57) contains two $J^P = 0^-$ pseudoscalar particles, its parity will be given by

$$P = (-1)^S \;, \qquad (6.3.59)$$

where S is its total angular momentum. Thus we see that if parity were conserved in the weak interaction, then the decay (6.3.57) would be forbidden. Since the kaon spin is even, the kaon must be a boson, so that its final state of two pions must have a wave function which is symmetric under the interchange of the two pions. We find that states with $I = 0$ and $I = 2$ fulfill this requirement. However, $I = 0$ is forbidden, since it would imply charge nonconservation, because it has zero charge, whereas the $K^{\pm}$ has nonzero charge. Thus the final state pions must have $I = 2$, $I_3 = 1$.

In the decay (6.3.58), we find that Dalitz plots for the emergent pion energies are roughly isotropic, indicating that the matrix element for $K_{3\pi}$

decay is effectively independent of pion energy (see 7.1), and suggesting zero spin for the kaon. In terms of isospin (5.1), we see that the final 3π state may have I = 0, 1, 2 or 3, and is not uniquely characterized by its total isospin, each of the pions having I = 1. Let $I^{(12)}$ denote the intermediate isospin of two of the pions. This quantity can take one of the three possible values: 0, 1 and 2. If the total isospin of the three-pion system ($I^{(123)}$) is to be zero, then we must have $I^{(12)} = 1$, $I^{(3)} = 1$. If the total isospin is to be 3, then evidently $I^{(12)} = 2$. For $I^{(123)} = 1$ or 2, there are number of different possible states, distinguished by differing values of $I^{(12)}$. As we have shown above, the three-pion system must have $I_3 = 1$. Thus there remain six possibilities $|T^{(12)}; T, T_3\rangle$:

$$|0; 1, 1\rangle = (1/\sqrt{3})\,(|\pi^+, \pi^-, \pi^0\rangle + |\pi^-, \pi^+, \pi^+\rangle - |\pi^0, \pi^0, \pi^+\rangle) \qquad (6.3.60)$$

$$|1; 1, 1\rangle = (1/2)\,(|\pi^+, \pi^0, \pi^0\rangle - |\pi^0, \pi^+, \pi^0\rangle - |\pi^+, \pi^-, \pi^+\rangle + |\pi^-, \pi^+, \pi^+\rangle) \qquad (6.3.61)$$

$$|2; 1, 1\rangle = (1/2\sqrt{15})\,(|\pi^+, \pi^-, \pi^+\rangle + |\pi^-, \pi^+, \pi^+\rangle + 2|\pi^0, \pi^0, \pi^+\rangle - 3|\pi^+, \pi^0, \pi^0\rangle - 3|\pi^0, \pi^+, \pi^0\rangle + 6|\pi^+, \pi^+, \pi^-\rangle) \qquad (6.3.62)$$

$$|1; 2, 1\rangle = (1/2)\,(|\pi^+, \pi^0, \pi^0\rangle - |\pi^0, \pi^+, \pi^0\rangle + |\pi^+, \pi^-, \pi^+\rangle - |\pi^-, \pi^+, \pi^+\rangle) \qquad (6.3.63)$$

$$|2; 2, 1\rangle = (1/2\sqrt{3})\,(2|\pi^+, \pi^+, \pi^-\rangle - 2|\pi^0, \pi^0, \pi^+\rangle + |\pi^+, \pi^0, \pi^0\rangle + |\pi^0, \pi^+, \pi^0\rangle - |\pi^+, \pi^-, \pi^+\rangle - |\pi^-, \pi^+, \pi^+\rangle) \qquad (6.3.64)$$

$$|2; 3, 1\rangle = (1/\sqrt{15})\,(2|\pi^+, \pi^0, \pi^0\rangle + 2|\pi^0, \pi^+, \pi^0\rangle + 2|\pi^0, \pi^0, \pi^+\rangle + |\pi^+, \pi^-, \pi^+\rangle + |\pi^-, \pi^+, \pi^+\rangle + |\pi^+, \pi^+, \pi^-\rangle) \qquad (6.3.65)$$

The multiplicative factors arise from Clebsch-Gordan coeffecients (see Appendix B). Of the isospin states (6.3.60) through (6.3.65), only one, on its own, has the symmetry properties required (i.e. invariance under pion interchange). This is (6.3.65). However, a linear combination of the states (6.3.60), (6.3.61) and (6.3.62) will also serve:

$$|S; 1, 1\rangle = (1/\sqrt{15})\,(2|\pi^+, \pi^-, \pi^-\rangle + 2|\pi^-, \pi^+, \pi^+\rangle + 2|\pi^+, \pi^+, \pi^-\rangle - |\pi^0, \pi^0, \pi^+\rangle - |\pi^+, \pi^0, \pi^0\rangle - |\pi^0, \pi^+, \pi^0\rangle) \qquad (6.3.66)$$

Thus, in general, the isospin form of the final three-pion state will be given by

$$|3\pi\rangle = a\,|2; 3, 1\rangle + b\,|S; 1, 1\rangle, \qquad (6.3.67)$$

where a and b are two complex coefficients. We find that the two possible charge states of the decay which we are investigating have a branching ratio given by

$$R = \frac{W(K \rightarrow \pi^+ + \pi^\circ + \pi^\circ)}{W(K \rightarrow \pi^+ + \pi^+ + \pi^-)} = \left|\frac{2a - b}{a + 2b}\right|^2 . \qquad (6.3.68)$$

The result (6.3.68) assumes that all the pions have exactly the same mass. However, this is not precisely the case, and in the low-energy reactions which we are considering here, the pion mass difference can have a significant effect. Calculating this effect using phase-space volumes (20), we obtain

$$R = 1.243 \left|\frac{2 - x}{1 + 2x}\right|^2 , \qquad (6.3.69)$$

where

$$x = b/a . \qquad (6.3.70)$$

As $|x| \longrightarrow \infty$, $R \longrightarrow 0.311$, and as $x \longrightarrow 0$, $R \longrightarrow 4.97$. Experimentally, the value for R is (21)

$$R = 0.29 \pm 0.04 , \qquad (6.3.71)$$

which is consistent with a large value for the parameter x. Thus it seems likely that the three-pion state is predominantly of the symmetric form (6.3.66), although a small admixture of the state (6.3.65) cannot be excluded. This implies that the final state of the decay (6.3.58) usually has

$$I = 1. \qquad (6.3.72)$$

6.4 Hyperon Decays.

Among the 'V-particles' mentioned in 6.3, there existed one particle, known as the Λ°, which, from momentum and energy measurements of its decay products, was found to have a mass of (22)

$$1115.60 \pm 0.05 \quad MeV/c^2 . \qquad (6.4.1)$$

The Λ° lifetime was established by measurement of the delay between production and decay in a bubble chamber as

$$(2.578 \pm 0.021) \times 10^{-10} \quad s. \qquad (6.4.2)$$

There was also a massive triplet, known as the Σ particles, which were found to be responsible for a number of the 'V-particle' tracks obtained from cosmic rays. By analysis of the proton range in its decay, the Σ^+ mass was ascertained as

$$1189.37 \pm 0.06 \quad MeV/c^2 , \qquad (6.4.3)$$

and its lifetime, from track angle and length measurements, as

$$(0.800 \pm 0.006) \times 10^{-10} \quad s . \qquad (6.4.4)$$

The Σ^- mass was found by measuring the ranges of the sigma particles in the reactions

$$K^- + p \longrightarrow \Sigma^+ + \pi^- , \qquad (6.4.5)$$

$$K^- + p \longrightarrow \Sigma^- + \pi^+ . \qquad (6.4.6)$$

From (6.4.5) and (6.4.6), the mass difference between the Σ^+ and the Σ^- was calculated, and, knowing the value of the Σ^+ mass (6.4.3), this yielded a value of

$$1197.35 \pm 0.06 \quad MeV/c^2 \qquad (6.4.7)$$

for the Σ^- mass. By similar methods to those employed for the Σ^+, the Σ^- lifetime was found to be

$$(1.482 \pm 0.017) \times 10^{-10} \quad s . \qquad (6.4.8)$$

By measuring the $\Sigma^+ - \Sigma^\circ$ mass difference, the Σ° mass becomes

$$1192.48 \pm 0.08 \quad MeV/c^2 , \qquad (6.4.9)$$

and its lifetime, by nuclear emulsion range measurements, is established as

$$< 1.0 \times 10^{-14} \quad s . \qquad (6.4.10)$$

Theoretical estimates for the Σ° lifetime give (23)

$$\sim 5 \times 10^{-17} \quad s . \qquad (6.4.11)$$

A further type of 'V-particle' was the cascade or Ξ hyperon, which decayed into another 'V-particle', which in turn decayed into stable particles. From track analysis in heavy-liquid bubble chambers, the Ξ^- mass was calculated as

$$1321.29 \pm 0.014 \quad MeV/c^2 , \qquad (6.4.12)$$

and its lifetime as

$$(1.652 \pm 0.023) \times 10^{-10} \quad s . \qquad (6.4.13)$$

However, the 'Strangeness Scheme' [4] of Gell-Mann and Nishijima (24) indicated that there should also exist a Ξ°. This was found using bubble chambers, and track analysis demonstrated that its mass was

$$1314.9 \pm 0.6 \quad MeV/c^2 , \qquad (6.4.14)$$

and that its lifetime was

$$(2.96 \pm 0.12) \times 10^{-10} \quad s . \qquad (6.4.15)$$

The last hyperon to be discovered was the Ω^-, which had been predicted by Gell-Mann (25) using SU(3) symmetry (see chapter 8). From the 41 bubble chamber events involving the Ω^- photographed to date, its tentative mass assignment is

$$1672.2 \pm 0.4 \quad \text{MeV}/c^2 , \tag{6.4.16}$$

and its lifetime is thought to be

$$(1.3 \pm 0.25) \times 10^{-10} \text{ s} . \tag{6.4.17}$$

We now tabulate the decays and branching ratios for the hyperons (26):

Particle	Decay Mode	Branching Ratio
Λ^0	$p\pi^-$	(64.2 ± 0.5) %
	$n\pi^0$	(35.8 ± 0.5) %
	$p e v$	$(8.13 \pm 0.29) \times 10^{-4}$
	$p \mu v$	$(1.57 \pm 0.35) \times 10^{-4}$
	$p \pi^- \gamma$	$(0.85 \pm 0.14) \times 10^{-3}$
Σ^+	$p\pi^0$	(51.6 ± 0.7) %
	$n\pi^+$	(48.4 ± 0.7) %
	$p\gamma$	$(1.24 \pm 0.18) \times 10^{-3}$
	$n\pi^+\gamma$	$(0.93 \pm 0.10) \times 10^{-3}$
	$\Lambda e^+ v$	$(2.02 \pm 0.47) \times 10^{-5}$
	$n \mu^+ v$	$(< 2.4) \times 10^{-5}$
	$n e^+ v$	$(< 1.0) \times 10^{-5}$
	$p e^+ e^-$	$(< 7) \times 10^{-6}$
Σ^0	$\Lambda\gamma$	(100) %
	$\Lambda e^+ e^-$	$(< 5.45) \times 10^{-3}$
Σ^-	$n\pi^-$	(100) %
	$n e^- v$	$(1.08 \pm 0.04) \times 10^{-3}$
	$n \mu^- v$	$(0.45 \pm 0.04) \times 10^{-3}$
	$\Lambda e^- v$	$(0.60 \pm 0.06) \times 10^{-4}$
	$n\pi^-\gamma$	$(1.0 \pm 0.2) \times 10^{-4}$
Ξ^0	$\Lambda\pi^0$	(100) %
	$p\pi^-$	$(< 0.9) \times 10^{-3}$
	$p e^- v$	$(< 1.3) \times 10^{-3}$
	$\Sigma^+ e^- v$	$(< 1.5) \times 10^{-3}$

Particle	Decay Mode	Branching Ratio
	$\Sigma^- e^+ v$	$(< 1.5) \times 10^{-3}$
	$\Sigma^+ \mu^- v$	$(< 1.5) \times 10^{-3}$
	$\Sigma^- \mu^+ v$	$(< 1.5) \times 10^{-3}$
	$p \mu^- v$	$(< 1.3) \times 10^{-3}$
Ξ^-	$\Lambda \pi^-$	(100) %
	$\Lambda e^- v$	$(0.70 \pm 0.21) \times 10^{-3}$
	$\Sigma^\circ e^- v$	$(< 0.5) \times 10^{-3}$
	$\Lambda \mu^- v$	$(< 1.3) \times 10^{-3}$
	$\Sigma^\circ \mu^- v$	(< 0.5) %
	$n \pi^-$	$(< 1.1) \times 10^{-3}$
	$n e^- v$	(< 1.0) %
Ω^-	$\Xi^\circ \pi^-$	Total of 41 events seen
	$\Xi^- \pi^\circ$	
	ΛK^-	

We now make a number of general comments concerning the hyperon decays, and discuss the decays of particular hyperons in greater detail in the following sections. For the beta decay

$$\Lambda^\circ \longrightarrow p + e^- + \bar{v}_e \quad , \tag{6.4.18}$$

we find that we are unable to write a simple four-term matrix element as we were able to for the neutron (5.5.1), (5.5.2), since we cannot now assume that vector current is conserved, and the weak magnetic form factor is no longer unimportant, since the decay (6.4.18) has significantly more disintegration energy than the neutron decay. Thus its matrix element becomes

$$M_{if} = G/\sqrt{2} \; (V_i + A_i)\, \bar{u}_e \gamma_i (1 + \gamma_5)\, u_v \quad , \tag{6.4.19}$$

where V_i and A_i are the vector and axial vector currents respectively, defined:

$$V_i = \bar{u}_p (f_1 \gamma_i + f_2 \sigma_{ij} q_j + f_3 q_i)\, u_\Lambda \quad , \tag{6.4.20}$$

$$A_i = \bar{u}_p (g_1 \gamma_i + g_2 \sigma_{ij} q_j + f_3 q_i)\, \gamma_5 u_\Lambda \quad , \tag{6.4.21}$$

where

$$q = p_\Lambda - p_p = p_e + p_\nu \quad . \tag{6.4.22}$$

Since we have six unknown form factors in (6.4.20) and (6.4.21), and only two equations, it is impossible to solve for them and hence to obtain a value for the rate of hyperon beta decay. However, we try assuming that the momentum transferred, q, is negligible, and hence

$$f_2 = f_3 = g_2 = g_3 = 0 \; . \tag{6.4.23}$$

Further we set

$$f_1 = g_1 = 1 \; , \tag{6.4.24}$$

although this has little justification. With the assumptions (6.4.23) and (6.4.24), the matrix element (6.4.19) reduces to

$$M_{if} = (G/\sqrt{2}) \; \bar{u}_p \gamma_i (1 + \gamma_5) u_\Lambda \bar{u}_e \gamma_i (1 + \gamma_5) u_\nu \; . \tag{6.4.25}$$

(6.4.25) gives

$$W = (G^2 D^5 C)/(15\pi^3) \; , \tag{6.4.26}$$

where D is the maximum electron energy:

$$D = (M_Y^2 - M_N^2)/2M_Y \; , \tag{6.4.27}$$

where M_Y is the hyperon mass and M_N the nucleon mass. In the expression for the transition rate (6.4.26), C is a dimensionless constant which compensates for the recoil of the hyperon and its decay products. When $D \longrightarrow 0$, $C \longrightarrow 1$, and when $D \longrightarrow M_Y/2$, $C \longrightarrow 2.5$, so that C is roughly constant for all decays. We note that for $C \longrightarrow 0$, the formula (6.4.26) becomes the neutron decay probability:

$$W_n = (G^2 D^5 (1 + 3a^2))/(60\pi^3) \; , \tag{6.4.28}$$

where a is the ratio of axial vector to vector coupling strengths, so that here we put a = 1. In the second limiting case $D = M_Y/2$, we find that (6.4.26) tends to

$$W' = (G^2 m^5)/(192\pi^3), \tag{6.4.29}$$

which is the muon decay rate, so that we set $m = m_\mu$. By means of the formula (6.4.26), we should be able to estimate the branching ratios for hyperon beta decays. Thus, for example, for the Λ° this should be 1.5 %, and for the Σ^- 5.8 %. However, experiments (27) indicate that the actual values for these branching ratios are

$$(8.13 \pm 0.29) \times 10^{-4} \tag{6.4.30}$$

and

$(1.08 \pm 0.04) \times 10^{-3}$ (6.4.31)

respectively, in complete disagreement with our theoretical predictions. Thus we are forced to conclude that one or both of the assumptions (6.4.23) and (6.4.24) was unjustified.

6.5 Lambda Hyperon Decay.

According to the general scheme of the weak interaction, it might be reasonable to expect that the nucleonic current

$$(\bar{n} \quad p) \tag{6.5.1}$$

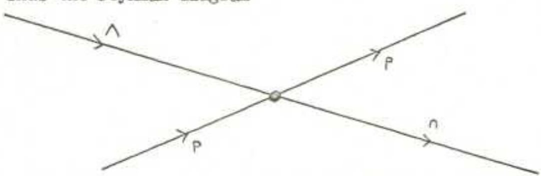
should interact not only with the leptonic current, but also with the strange current[5]

$$(\bar{\Lambda} \quad p) \; . \tag{6.5.2}$$

The interaction of the two currents (6.5.1) and (6.5.2) has the form

$$(\bar{n} \quad p)(\bar{\Lambda}\, p)^{+} + (\bar{\Lambda}\, p)(\bar{n} \quad p)^{+} = (\bar{n} \quad p)(\bar{p}\, \Lambda) + (\bar{\Lambda}\, p)(\bar{p} \quad n) \tag{6.5.3}$$

Thus the Feynman diagram



(6.5.4)

corresponds to the first term in this interaction. However, by transposing an incoming particle for an outgoing antiparticle, the simple scattering process (6.5.4) becomes

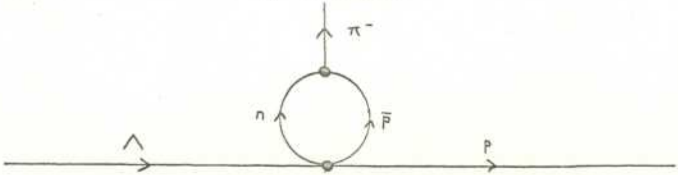
$$\Lambda \longrightarrow \bar{p} + n + p \; . \tag{6.5.5}$$

The Λ° decays

$$\Lambda^{\circ} \longrightarrow p + \pi^{-} \quad , \tag{6.5.6}$$

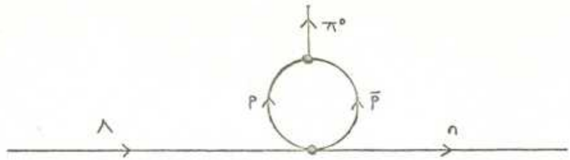
$$\Lambda^{\circ} \longrightarrow n + \pi^{\circ} \quad , \tag{6.5.7}$$

may thus be accounted for by the diagrams



(6.5.8)

and



(6.5.9)

respectively. Alternatively, we could have drawn a closed p $\bar{p}$ loop, so that the Λ becomes a virtual neutron, which becomes real with the emission of a pion to conserve momentum and energy.

We now consider the asymmetry parameters governing the angular distribution of polarized Λ° decay products. We assume throughout that the Λ° has a spin of $\frac{1}{2}$, and there exists strong experimental evidence to reinforce this view (28). We rewrite the matrix element (6.4.19) in the form

$$M_{if} = (1/\sqrt{2 E_{\pi}}) \; \bar{u}_N (\underline{p}_n) (1 + \rho \gamma_5) u_{\Lambda} (\underline{p}_{\Lambda}) F \cdot \int d^3x \exp (j(p_{\Lambda} - p_N - p_{\pi})x) \quad (6.5.10)$$

where F is a scalar amplitude and ρ is a parameter describing the parity-violating component of the interaction. At first sight, (6.5.10) appears to describe only a pure scalar interaction. However, rewriting the middle factor (containing the spinors) of (6.5.10) for a vector interaction, we obtain

$$M' = j \, \bar{u}_N(\underline{p}_N) \gamma_r (1 + \rho \gamma_5) u_{\Lambda}(\underline{p}_{\Lambda}) ((p_N)_r F_1 + (p_{\pi})_r F_2 + (p_{\Lambda})_r F_3)) \quad (6.5.11)$$

which is essentially the same as (6.4.20). In (6.5.11) we have three form factor constants. However, we may immediately write the term in F_2 without the factor F_2 using momentum conservation in terms of p_N and p_{Λ}. Using the Dirac equation we find (29) that

$$F' = (F_2 - F_1)M_N - (F_2 + F_3)M_{\Lambda} , \quad (6.5.12)$$

$$F' \rho' = ((F_2 - F_1)M_N + (F_2 + F_3)M_{\Lambda})\rho , \quad (6.5.13)$$

where F' is our new single amplitude. Obviously this may be determined uniquely by the equations (6.5.12) and (6.5.13), and thus we see that the vector matrix element (6.5.11) is completely equivalent to the scalar one (6.5.10). It may be shown that all other permitted types of interactions also produce matrix elements equivalent to (6.5.10).

From (6.5.10), we may now write the transition rate for Λ° decay as

$$(\delta W/\delta t) = (|F|^2/8\pi)\,(d\Omega_\pi/4\pi)\,(1 + ((M_N^2 - m_\pi^2)/M_\Lambda^2))\ \times$$
$$\times\ (1/M_\Lambda)\ \sqrt{\lambda(M_\Lambda^2, M_N^2, m_\pi^2)}\ \left|\bar{u}_N(p_N)(1+\rho\gamma_5)\,u_\Lambda(0)\right|^2 \tag{6.5.14}$$

where λ is an arbitrary function such that

$$|p_N| = |p_\pi| = (1/2M_\Lambda)\ \sqrt{\lambda(M_\Lambda^2, M_N^2, m_\pi^2)}\ . \tag{6.5.15}$$

In (6.5.14) we have integrated over all angles except for the angle which the Λ° polarization vector makes with the direction of motion of the pion. Denoting this quantity by Θ , we obtain the angular distribution:

$$W(\Theta)\ d(\cos\Theta) = \left|\bar{u}_N(p_N)\ (1 + \rho\gamma_5)\ u_\Lambda(0)\right|^2\ d(\cos\Theta)\ . \tag{6.5.16}$$

Since the spinor $u_\Lambda(0)$ vanishes except for the component with Dirac index one, which is unity, we may rewrite (6.5.16):

$$W(\Theta) = \left|(\bar{u}_N(p_N)\ (1 + \rho\gamma_5))_1\right|^2\ . \tag{6.5.17}$$

Thus we find that, for the decay of totally polarized Λ particles:

$$W(\Theta) = k\ .\ (1 + a\cos\Theta)\ , \tag{6.5.18}$$

where

$$a = (2\,|p_N|\ \mathrm{Re}(\rho)\,)/(1 + |\rho|^2\ |p_N|^2/(M_N + E_N)^2\,)\ . \tag{6.5.19}$$

If the angular distribution (6.5.18) is to be isotropic, then we see that a must vanish, corresponding to a zero or infinite value for the parity-violation parameter ρ . In these cases, parity is said to be conserved. Thus a nonisotropic angular distribution implies parity violation. However, the angular distribution is always isotropic when the original Λ is unpolarized.

Experimentally, Λ particles produced in the reaction

$$\pi^- + p \longrightarrow \Lambda + K^0\ , \tag{6.5.20}$$

known as 'associated production' (30), tend to be polarized perpendicularly to the $p\pi^-$ scattering plane. Observations of Λ particles produced in (6.5.20) indicate that the product (31)

$$a\,P = 0.55 \pm 0.06 \tag{6.5.21}$$

for

$$\Lambda \longrightarrow \pi^- + p\ , \tag{6.5.22}$$

and

$$a\,P = 0.60 \pm 0.13 \tag{6.5.23}$$

for

$$\Lambda \longrightarrow \pi^\circ + n\ . \tag{6.5.24}$$

It should be noted that we are never able to meaure the parameter a on its own, but only in the product aP, P being the polarization of the Λ°. Furthermore, since we are never able to determine P, our values for aP, (6.5.21) and (6.5.23) only set a lower limit upon a, and leave it unsigned.

Slightly more information may be obtained by observing the asymmetry in the angular distribution of the nucleons, rather than the pions, in Λ decay. We define the initial Λ polarization by

$$\underline{P} \quad = \quad (W_+ \; - \; W_-)\underline{e}_z \; , \tag{6.5.25}$$

where W_+ is the number of particles with their spins aligned in the + z direction, and W_- the number in the - z direction, and $\underline{e}_z$ is the unit vector in the z direction. Obviously

$$W_+ + W_- \quad = \quad 1 \; . \tag{6.5.26}$$

Since the spin of the Λ is $\frac{1}{2}$, and it is thought that total angular momentum is always conserved, the final two-particle state resulting from Λ decay must be either S- or P-wave, i.e. it must have an orbital angular momentum of 0 or 1. Using spherical harmonics[6], we find that

$$|\psi_S\rangle \quad = \quad A_S\,\xi_+ \quad , \tag{6.5.27}$$

$$|\psi_P\rangle \quad = \quad A_P\,(\xi_+ \cos\theta + \xi_- \sin\theta\, e^{j\phi}\;) \; , \tag{6.5.28}$$

omitting the normalization factor, where A_P is the transition amplitude for a P-wave state, and $\xi_\pm$ represents the sense of polarization. In our case, we find that when the Λ is polarized in a + z direction

$$|\psi_f^+\rangle \;=\; (A_S \;+\; A_P \cos\theta)\xi_+ \;+\; A_P\, e^{j\phi} \;\sin\theta\,\xi_- \; , \tag{6.5.29}$$

and when in a - z direction

$$|\psi_f^-\rangle \;=\; A_P\, e^{-j\phi}\;\sin\theta\,\xi_+ + (A_S \;-\; A_P \cos\theta)\xi_- \; . \tag{6.5.30}$$

Thus the expectation value of the component of the nucleon spin operator in the final state is given by

$$\langle\psi_f^+\,|\,\sigma_z\,|\,\psi_f^+\rangle \;=\; |A_S \;+\; A_P \cos\theta|^2 \;-\; |A_P|^2 \sin^2\theta \, , \tag{6.5.31}$$

$$\langle\psi_f^-\,|\,\sigma_z\,|\,\psi_f^-\rangle \;=\; |A_P|^2 \sin^2\theta \;-\; |A_S \;-\; A_P \cos\theta\,|^2 \, . \tag{6.5.32}$$

Substituting with (6.5.25) for the polarization of the initial Λ, we obtain

$$\langle\sigma_z\rangle \quad = \quad 2\,\mathrm{Re}(A_P\, A_S^*)\cos\theta + P(|A_S|^2 \;+\; |A_P|^2\cos(2\theta)\,)\; . \tag{6.5.33}$$

The other two components of the nucleon spin may be calculated in a similar manner. Introducing the unit vectors $\underline{e}_\pi$ and $\underline{e}_p$, and using a vector notation for the polarization $\underline{P}$, we obtain

$$\langle \underline{\sigma} \rangle = 2\,\mathrm{Re}(A_P A_S^*)\underline{e}_\pi - 2\,\mathrm{Im}(A_P A_S^*)\underline{e}_\pi \times \underline{P} + (|A_S|^2 - |A_P|^2)\,\underline{P} + 2|A_P|^2\,\underline{e}_\pi(\underline{e}_\pi \underline{P}) \quad . \qquad (6.5.34)$$

The result (6.5.34) is usually written, in experimental work, as

$$\langle \underline{\sigma} \rangle = k\,.\,(-(\alpha - \underline{e}_p\,\underline{P})\,\underline{e}_p + \beta\,\underline{e}_p \times \underline{P} + \gamma((\underline{e}_p \times \underline{P}) \times \underline{e}_p)) \quad , \qquad (6.5.35)$$

where

$$\alpha = 2\,(\mathrm{Re}(A_P A_S^*))/(|A_P|^2 + |A_S|^2) \quad , \qquad (6.5.36)$$

$$\beta = 2\,(\mathrm{Im}(A_P A_S))/(|A_P|^2 + |A_S|^2) \quad , \qquad (6.5.37)$$

$$\gamma = (|A_S|^2 - |A_P|^2)/(|A_S|^2 + |A_P|^2) \quad , \qquad (6.5.38)$$

so that

$$\alpha^2 + \beta^2 + \gamma^2 = 1 \quad . \qquad (6.5.39)$$

The constant k in (6.5.35) is usually taken as

$$k = 1/(1 - \alpha\,\underline{e}_p\,\underline{P}) \qquad (6.5.40)$$

in order to normalize the expectation value operator of the nucleon spin. Recalling 3.3 we find that T invariance of the Hamiltonian (6.5.10) demands ρ to be real, so that β vanishes. In the case where the initial Λ particles are unpolarized, i.e. $\underline{P} = 0$, we see that (6.5.35) simplifies to

$$\langle \underline{\sigma} \rangle = -\alpha\,\underline{e}_p \quad . \qquad (6.5.41)$$

Thus the protons from unpolarized Λ decay are longitudinally polarized by the amount $-\alpha$. Hence we have found a method of obtaining a value for α on its own. Experiments on the decay (6.5.22) give (32)

$$\alpha = 0.647 \pm 0.016 \quad , \qquad (6.5.42)$$

$$\beta = -0.10 \pm 0.07 \quad , \qquad (6.5.43)$$

$$\gamma = 0.75 \pm 0.02 \quad . \qquad (6.5.44)$$

Although β does not vanish within experimental error (6.5.43), due to a low amplitude interaction between the final pion and proton, a small value of β might be expected, so that T invariance is not forbidden. We note that the experimental values (6.5.42), (6.5.43) and (6.5.44) obey the condition (6.5.39) within experimental error. We see that the ratio of the amplitudes A_P and A_S is real, and that numerically it is

$$A_P/A_S = -0.36 \pm 0.05 \quad , \qquad (6.5.45)$$

so that the P-wave amplitude is about three times as strong as the S-wave one. We note that, substituting the result (6.5,42) in (6.5.21), we obtain P = 1, and thus the Λ° particles from the reaction (6.5.20) are almost completely polarized. The asymmetry parameters α, β, and γ cannot be measured directly in the decay (6.5.24), but assuming that the polarization of the initial Λ particles was the same for (6.5.22) as for (6.5.24), experiments show that (33)

$$\frac{\alpha\,(\Lambda \longrightarrow n + \pi^{\circ})}{\alpha\,(\Lambda \longrightarrow p + \pi^{-})} = 1.10 \pm 0.27 , \qquad (6.5.46)$$

so that the ratio is unity within experimental error.

6.6 Sigma Hyperon Decay.

The formula (6.5.36) presented above is, in fact, valid for any spin $\frac{1}{2}$ particle. Since there is good evidence (34) to support the view that the sigma particles have spin $\frac{1}{2}$, we may also apply it to their decay. However, as was seen in 6.4, the Σ multiplet possess three dominant weak decay modes:

$$\Sigma^{+} \longrightarrow p + \pi^{\circ} , \qquad (6.6.1)$$

$$\Sigma^{+} \longrightarrow n + \pi^{+} , \qquad (6.6.2)$$

$$\Sigma^{-} \longrightarrow n + \pi^{-} . \qquad (6.6.3)$$

It is customary to distinguish between the amplitudes of these decays by writing the sign of the pion in the final state of the decay as a subscript to the amplitude. Assuming the reality of the amplitudes and hence T invariance, we may write (6.5.36) as

$$\alpha_{i} = 2(A_{P}^{i}\, A_{S}^{i})/(|A_{P}^{i}|^{2} + |A_{S}^{i}|^{2}) , \qquad (6.6.4)$$

where the subscript i may be o, - or + . The parameter α_{o} has been measured directly (35):

$$\alpha_{o} = 0.78 \pm 0.08 . \qquad (6.6.5)$$

We note that within experimental error, the parameter (6.6.5) has the same magnitude but a different sign from that for the Λ° (6.5.42).

The parameters α_{+} and α_{-} may not, however, be measured directly, since it is not possible to measure the polarization of the neutron in (6.6.2) and (6.6.3) in the same manner as the polarization of the proton is measured in (6.6.4). However, comparison of the asymmetry in the pions from (6.6.2) and (6.6.1) yields (36)

$$(\alpha_+)/(\alpha_o) = (\alpha_+ P)/(\alpha_o P) = (0.03 \pm 0.08)/(0.75 \pm 0.17) =$$
$$= 0.04 \pm 0.11 , \qquad (6.6.6)$$

and, using the known value for α_o (6.6.5), we obtain

$$\alpha_+ = 0.03 \pm 0.09 . \qquad (6.6.7)$$

Thus, within experimental error, the value of α_+ vanishes, demonstrating that parity is conserved in the decay (6.6.2). Experiments on the resonance K(1520) indicate that (37)

$$\alpha_- = 0.13 \pm 0.16 , \qquad (6.6.8)$$

again favouring parity conservation. Thus it appears that in the decays (6.6.2) and (6.6.3), the parity-violating amplitude is small if it exists at all. These decays afford some of the only examples of non-parity-violating weak interactions.

We now consider the amplitudes A_P and A_S present in Σ decay. Using a suitable normalization factor, and ignoring the difference in phase-space factors due to the $\Sigma^+ - \Sigma^-$ mass difference, we have

$$1/T_{\Sigma^-} = (A_S^-)^2 + (A_P^-)^2 , \qquad (6.6.9)$$

$$1/T_{\Sigma^+} = (A_S^+)^2 + (A_P^+)^2 + (A_S^o)^2 + (A_P^o)^2 , \qquad (6.6.10)$$

assuming time-reversal invariance. Approximating (6.6.7) by

$$\alpha_+ = 0 , \qquad (6.6.11)$$

we see that either A_P or A_S vanishes. We arbitrarily choose

$$A_P = 0 . \qquad (6.6.12)$$

We know that

$$R(\Sigma^+) = ((A_S^o)^2 + (A_P^o)^2)/((A_S^+)^2 + (A_P^+)^2 + (A_S^o)^2 + (A_P^o)^2) \qquad (6.6.13)$$

and experiments demonstrate that (38)

$$R(\Sigma^+) = \pm 0.50 \pm 0.02 , \qquad (6.6.14)$$

and hence

$$(A_S^o)^2 + (A_P^o)^2 = (A_S^+)^2 = 1/2T_{\Sigma^+} . \qquad (6.6.15)$$

We now wish to determine A_P^o and A_S^o in terms of α_o. However, since the relation (6.6.4) is quadratic, we obtain two possible values for each amplitude:

$$A_S^o = \pm(0.43 \pm 0.05)\, A_S^+ , \qquad (6.6.16)$$

$$A_P^{o} = (0.90 \pm 0.03)\, A_S^{+} \, , \qquad (6.6.17)$$

or

$$A_S^{o} = (0.43 \pm 0.05)\, A_S^{+} \, , \qquad (6.6.18)$$

$$A_P^{o} = (0.90 \pm 0.03)\, A_S^{+} \, , \qquad (6.6.19)$$

where

$$A_S^{+} = 1/\sqrt{2T_{\Sigma^+}} \, . \qquad (6.6.20)$$

Approximating (6.6.8) by

$$\alpha_- = 0 \, , \qquad (6.6.21)$$

we obtain

$$A_S^{-} = 0 \, , \qquad (6.6.22)$$

$$A_P^{-} = \pm 1/\sqrt{T_{\Sigma^-}} \sim \pm A_S \, , \qquad (6.6.23)$$

or

$$A_P^{-} = 0 \, , \qquad (6.6.24)$$

$$A_S^{-} = \pm 1/\sqrt{T_{\Sigma^-}} \sim \pm A_S^{+} \, . \qquad (6.6.25)$$

Clearly we have a considerable choice of values for the amplitudes. We shall use the results obtained above in the next section.

6.7 Isotopic Selection Rules in Hyperon Decays.

In (5.3.23) we mentioned the partial conservation law

$$|\Delta I| = \tfrac{1}{2} \, . \qquad (6.7.1)$$

However, examining our expression for the strange current - nucleonic current interaction (6.5.3), we see that we can only definitely write

$$|\Delta I| = \tfrac{1}{2} \, , 3/2 \, . \qquad (6.7.2)$$

We know the Gell-Mann - Nakano - Nishijima (GNN) relation (39)

$$Q = I_3 + Y/2 \, , \qquad (6.7.3)$$

and hence (6.7.2) corresponds to

$$\Delta Y = 1 \, , 2 \, . \qquad (6.7.4)$$

As we mentioned in 5.3, there exist a number of decays, which, if

$$\Delta Y = 2 \qquad (6.7.5)$$

were allowed, should have much larger branching ratios than have been observed. Examples are

$$\Xi^{o} \longrightarrow p + \pi^{-} \qquad (6.7.6)$$

$\Omega^- \longrightarrow \Lambda^0 \pi^-$, (6.7.7)

$\Sigma^+ \longrightarrow n\, e^+\, \nu$. (6.7.8)

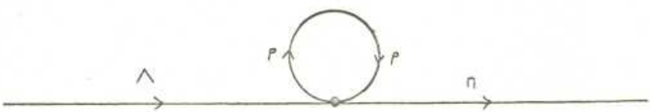
However, no decay with (6.7.5) has yet been observed, and so we are forced to conclude that (6.7.1) is a justified selection rule. If this is the case, then there should exist some theoretical basis for (6.7.1). At present, there are two separate hypotheses to account for (6.7.1). The first (40) is that the hadronic weak interactions are all caused by products of the charged current terms

$(\bar{n} \quad p)$ (6.7.9)

and

$(\bar{\Lambda} \quad p)$, (6.7.10)

so that (6.7.1) is an inherent property of the weak interaction. Transitions of the form (6.7.5) might be caused by the intervention of virtual strong interactions such as



(6.7.11)

The second theory is that the currents (6.7.9) and (6.7.10) are supplemented by the neutral hadron currents

$(\bar{\Lambda} \quad n)$, (6.7.12)

$(\bar{n} \quad n)$, (6.7.13)

$(\bar{p} \quad p)$. (6.7.14)

This would mean that all weak interactions, such as those of the form

$\bar{\Lambda} n\, (\bar{p}\, p \; + \; \bar{n}\, n)$, (6.7.15)

would obey (6.7.1) automatically. Recently, convincing evidence in favour of neutral hadron currents has been found in neutrino-hadron interactions, and we shall discuss this in more detail in the next section.

The final states in the Λ decays (6.5.6) and (6.5.7) may have either $I = \frac{1}{2}$ or $I = 3/2$ depending upon the isospin projections of the pion and nucleon. For the final state

$\pi^- + p$, (6.7.16)

Clebsch-Gordan coefficients give the coefficient of the $I = \frac{1}{2}$ wave function as $(\sqrt{2}/\sqrt{3})$, and of the $I = 3/2$ one as $(1/\sqrt{3})$. Similarly, the state

$$\pi^{\circ} + n \tag{6.7.17}$$

has a coefficient of $(-1/\sqrt{3})$ for the $I = \frac{1}{2}$ component of its wave function, and of $(\sqrt{2}/\sqrt{3})$ for its $I = 3/2$ component. Thus, superposing the $I = \frac{1}{2}$ components of the states (6.7.16) and (6.7.17), we obtain

$$\psi_{I=\frac{1}{2}} = (\sqrt{2}/\sqrt{3})(\pi^{-} + p) - (1/\sqrt{3})(\pi^{\circ} + n), \tag{6.7.18}$$

and similarly, for the $I = 3/2$ components:

$$\psi_{I=3/2} = (1/\sqrt{3})(\pi^{-} + p) + (\sqrt{2}/\sqrt{3})(\pi^{\circ} + n) . \tag{6.7.19}$$

The relation (6.7.18), assuming that the πN final state always has $I = \frac{1}{2}$, predicts the ratio of amplitudes for the decays (6.5.22) and (6.5.24) to be

$$\sqrt{2} : 1 , \tag{6.7.20}$$

so that the ratio of probabilities for the decays is

$$2 : 1 . \tag{6.7.21}$$

From (6.7.21) we may write

$$B = \frac{W(\Lambda \longrightarrow p + \pi^{-})}{W(\Lambda \longrightarrow n + \pi^{\circ}) + W(\Lambda \longrightarrow p + \pi^{-})} = \frac{2}{3} , \tag{6.7.22}$$

where B is the branching ratio for the decay (6.5.22). The experimental value for B is

$$B = 0.663 \pm 0.014 , \tag{6.7.23}$$

in excellent agreement with the prediction (6.7.22). Since the amplitudes for the decays (6.5.22) and (6.5.24) are similar (6.7.20), the angular correlations of the decay products must also be the same. Thus, by C invariance, the asymmetry parameter a must be the same for both decays. From (6.5.21) and (6.5.23) we see that experiments give (41)

$$a_0/a_- = 1.10 \pm 0.27 , \tag{6.7.24}$$

in good agreement with our prediction. A further check on the hypothesis (6.7.1) is afforded by studying the π° and π^{-} decay rates of the hyperfragment $_{\Lambda}He^4$. From (6.7.20), we may predict

$$A_p^{\circ}/A_S^{\circ} = 0.39 \pm 0.12 , \tag{6.7.25}$$

in good agreement with the experimental value of (42)

$$0.38 \pm 0.01 . \tag{6.7.26}$$

Returning to Σ decay, we now see that we may write, using Clebsch-Gordan coefficients,

$$|p, \pi^0\rangle \quad (\sqrt{2}/\sqrt{3}) \; |(3/2), \tfrac{1}{2}\rangle \; - \; (1/\sqrt{3}) \; |\tfrac{1}{2}, \tfrac{1}{2}\rangle \; , \qquad (6.7.27)$$

$$|n, \pi^+\rangle \quad (1/\sqrt{3}) \; |(3/2), \tfrac{1}{2}\rangle \; - \; (\sqrt{2}/\sqrt{3}) \; |\tfrac{1}{2}, \tfrac{1}{2}\rangle \; , \qquad (6.7.28)$$

$$|n, \pi^-\rangle \quad |(3/2), -(3/2)\rangle \; , \qquad (6.7.29)$$

adopting the convention $|I, I_3\rangle$ for the kets on the right-hand side. From (6.7.27), (6.7.28), and (6.7.29) we find that we may write

$$A_j^0 \quad = \quad (\sqrt{2/3})\, x \; - \; y/\sqrt{3} \quad , \qquad (6.7.30)$$

$$A_j^+ \quad = \quad x/3 \; + \; (\sqrt{2}/\sqrt{3})\, y \quad , \qquad (6.7.31)$$

$$A_j^- \quad = \quad x \quad , \qquad (6.7.32)$$

where j denotes the spin index, and may be either P or S. From (6.7.30), (6.7.31) and (6.7.32), we may eliminate x and y to obtain

$$X_j \quad = \quad A_j^+ \; + \; 2\, A_j^0 \; - \; A_j^- \quad = \quad 0 \; . \qquad (6.7.33)$$

(6.7.33) holds if and only if (6.7.1) is valid. Now we wish to find some combination of the amplitudes (6.6.16) through (6.6.25) which will fit the selection rule (6.7.33). We see that the choice (6.6.24), (6.6.25) violates (6.7.33) for $j = 1$, because of the fact that neither (6.6.17) nor (6.6.19) vanishes, and we chose

$$A_P^+ \quad = \quad 0 \; . \qquad (6.7.34)$$

Thus (6.6.22), (6.6.23) must be the correct choice. This yields

$$X_S \quad = \quad (0.39 \; \pm \; 0.07) A_S^+ \qquad (6.7.35)$$

if (6.6.16) and (6.6.17) hold, and

$$X_S \quad = \quad (-0.27 \; \pm \; 0.04) A_S^+ \qquad (6.7.36)$$

if (6.6.18) and (6.6.19) hold, taking the most favourable combination of signs. Similarly

$$X_P \quad = \quad (-0.27 \; \pm \; 0.04) A_P^+ \qquad (6.7.37)$$

or

$$X_P \quad = \quad (0.39 \; \pm \; 0.07) A_P^+ \, . \qquad (6.7.38)$$

Thus X_j does not vanish with experimental error, implying that (6.7.1) is not precisely true. If we interpret the amplitudes A^+, A^0, and A^- as vectors which have S and P components, then (6.7.33) implies that these should form a triangle, which is not, in fact, the case (43). The result (6.7.33) might also have been obtained by assuming the existence of an imaginary particle in sigma decay known as a 'spurion', with $I = \frac{1}{2}$, $I_3 = \frac{1}{2}$, $Y = 1$. This would imply no violation of isospin conservation in Σ decay. The 'spurion' approach is employed in Okun': Weak Interaction of Elementary Particles, Pergamon 1965, pp. 177-180.

Finally, we consider the predictions which may be made using isospin concerning the Ξ and Ω^- decays. From Clebsch-Gordan coefficients we see immediately that the ratio of the amplitudes in the decays

$$\Xi^- \longrightarrow \Lambda + \pi^-, \tag{6.7.39}$$

$$\Xi^0 \longrightarrow \Lambda + \pi^0 \tag{6.7.40}$$

is $(\sqrt{2})$: 1, and hence we predict

$$\frac{W(\Xi^- \longrightarrow \Lambda + \pi^-)}{W(\Xi^0 \longrightarrow \Lambda + \pi^0)} = 2 . \tag{6.7.41}$$

The experimental value for (6.7.41) is (44)

$$1.68 \pm 0.23 , \tag{6.7.44}$$

in agreement with our prediction. Similarly, all asymmetry parameters in the decays (6.7.39) and (6.7.40) should be equal, and experiments show that (45)

$$\alpha^{\Xi^-}/\alpha^{\Xi^0} = 1.22 \pm 0.50 . \tag{6.7.45}$$

We may use the same ratios and principles in Ω^- decay as in Ξ decay, and thus

$$\frac{W(\Omega^- \longrightarrow \Xi^0 + \pi^-)}{W(\Omega^- \longrightarrow \Xi^0 + \pi^-) + W(\Omega^- \longrightarrow \Xi^- + \pi^0)} = \frac{2}{3} , \tag{6.7.46}$$

and

$$\alpha^-/\alpha^0 = 1 . \tag{6.7.47}$$

However, due to the fact that very few Ω^- decays have been observed, because $S_{\Omega^-} = -3$, meaning that the Ω^- is only very rarely produced, no experimental values for (6.7.46) and (6.7.47) have yet been obtained.

6.8 Neutrino-Hadron Interactions.

We first discuss the hypercharge-conserving neutrino-hadron processes. With the restriction

$$\Delta Y = 0 , \tag{6.8.1}$$

and the assumption of nucleon targets, we have two elastic reactions:

$$\nu_1 + n \longrightarrow 1^- + p , \tag{6.8.2}$$

$$\bar{\nu}_1 + p \longrightarrow 1^+ + n , \tag{6.8.3}$$

and two inelastic ones:

$$v_1 + n \longrightarrow l^- + C , \qquad (6.8.4)$$

$$\bar{v}_1 + p \longrightarrow l^+ + C' , \qquad (6.8.5)$$

where C is any complex of strongly-interacting particles with Y = 1. The Hamiltonian for these processes (6.8.2), (6.8.3), (6.8.4) and (6.8.5) is

$$H = (G/\sqrt{2})\,(J_r(x) + L_r(x))\bar{L}_r(x) + \text{Herm. conj.}, \qquad (6.8.6)$$

where the leptonic current $L_r(x)$ is defined

$$L_r(x) = \sum_{l=e,\mu} j\,\bar{\Psi}_{v_l}\,\gamma_r(1 + \gamma_5)\Psi_l \qquad (6.8.7)$$

and

$$\bar{L}_r(x) = L_r(x)(1 - 2\delta_{r4}) . \qquad (6.8.8)$$

The Hamiltonian (6.8.6) assumes a local current-current form for the weak interaction, and uses the V-A theory, the two-component theory of the neutrino, and the conservation of leptons. We know very little indeed about the hadronic weak current $J_r(x)$, and the matrix elements of this current are interpreted as form factors, which are dependent upon strong interaction dynamics. However, the leptonic current (6.8.7) contains no form factors, and it is this which causes its 'local' or point property. At high energies, the interaction is dominated by the hadronic current, so that the neutrino cross-section is dependent purely upon the form factors:

$$(d\sigma_v)/(dq^2) = ((G^0)^2/2\pi)\,(|g_A(q^2)|^2 + |g_V(q^2)|^2 + q^2|f_V(q^2)|^2 + \\ + q^2|h_A(q^2)|^2) . \qquad (6.8.9)$$

An important question which has probably been settled by studying neutrino-hadron interactions is whether neutral lepton currents exist. We mentioned neutral currents in 4.4, and said that terms of the type (4.4.11), (4.4.12) and (4.4.13), at least in their pure leptonic form, were probably not present in the weak Hamiltonian. However, such semileptonic reactions as

$$v_\mu + p \longrightarrow v_\mu + n + \pi^+ , \qquad (6.8.10)$$

still involve no change in lepton or hadron current. In 1973, tracks representing the reaction (6.8.10) were obtained (46) in a liquid hydrogen bubble chamber. Further, a second neutral current reaction,

$$v_\mu + p \longrightarrow v_\mu + p + \pi^0 , \qquad (6.8.11)$$

was also observed. However, the rate for (6.8.10) and (6.8.11) has been shown to be less than 10% (47) of the rate for

$$\nu_\mu + n \longrightarrow \mu^- + p \ , \qquad (6.8.12)$$

suggesting that the charged current terms may have a larger amplitude than the neutral ones. However, these results are, as yet, only very tentative, and thus we may not say with any degree of certainty, that we must introduce a second leptonic coupling constant into the weak interaction.

We now consider briefly the so-called 'neutrino flip' hypothesis. There is no reason to assume that the lepton currents $(\bar{\nu}_e e)$ and $(\bar{\nu}_\mu \mu)$ are coupled to both the $\Delta Y = 0$ and the $\Delta Y = 1$ hadron currents. Hence it has been suggested that, instead, it is the $(\bar{\nu}_\mu e)$ and $(\bar{\nu}_e \mu)$ currents which are coupled to the $\Delta Y = 1$ hadron current (48). This hypothesis was known as the 'neutrino flip' theory because it interchanged the roles of the two neutrinos in hypercharge-changing semileptonic reactions. However, high-energy experiments show that the neutrinos arising from kaon decay, which the neutrino flip theory predicts to be electron neutrinos, produce muons when they interact with nucleons via $\Delta Y = 0$ currents, and not electrons, as the neutrino flip hypothesis demands. The result of a number of experiments (49) demonstrates that, if a neutrino flip coupling does exist, then its amplitude must be less than 20 % of the amplitude for the unflipped coupling. Thus the neutrino flip hypothesis, in the form given above, appears to be unlikely.

We now discuss the methods for confirming CP and hence T invariance in neutrino-hadron reactions. CP violation would be revealed by polarization in the final state nucleon from an unpolarized target. We consider the reaction (6.8.2) and we find that the final state transverse polarization P^t is given by

$$(d\sigma_\nu)/(dq^2)\, P^t \quad = \quad (F\underline{n} \cdot \underline{v} \times \underline{p})(v\,|\underline{p}|\,\sin\phi_\nu) \ , \qquad (6.8.13)$$

where $\underline{v}$ and $\underline{p}$ are the momentum vectors of the neutrino and proton respectively, v is the incident neutrino energy, $\underline{n}$ is the unit vector in the direction of the nucleon polarization, and ϕ_ν is the angle between $\underline{v}$ and $\underline{p}$, i.e. the proton recoil angle. F is a measure of the transverse polarization, and is a function of the form factors. Unfortunately, it is difficult to find transverse polarization due to the weak interaction in (6.8.2), since electromagnetic effects with a much higher amplitude also produce polarization. T invariance would cause the coupling constants affecting the form factors in F to be real, so that F would vanish, resulting in no weak interaction polarization. A second method of detecting T violation is to study a reaction of the type (50)

$$v_1 + Z \longrightarrow l^- + C \,, \tag{6.8.14}$$

where Z is an atomic nucleus and C is a hadron complex, for fixed lepton energy and fixed lepton-neutrino angle. Since the polarization of the lepton involves a factor

$$\underline{n} \cdot \underline{v} \times \underline{l} \,, \tag{6.8.15}$$

and since $\underline{n}$, the unit vector in the direction of the lepton polarization vector, changes sign under the operator T, sizeable lepton polarization would imply T violation. Again, no sensitive experiments have yet been carried out on the reaction (6.8.14).

The selection rule

$$\Delta I = 1 \,, \tag{6.8.16}$$

implies that J^o_r and $\bar{J}^o_r$ transform as pure isovector operators. The consequences of (6.8.16) may be tested in such processes as

$$v_1 + p \longrightarrow l^- + p + \pi^+, \tag{6.8.17}$$

$$v_1 + n \longrightarrow l^- + n + \pi^+, \tag{6.8.18}$$

$$v_1 + n \longrightarrow l^- + p + \pi^o . \tag{6.8.19}$$

However, since leptons have no isospin, the consequences of (6.8.16) in (6.8.17), (6.8.18) and (6.8.19) are the same as the consequences of isospin conservation in

$$S^+ + p \longrightarrow p + \pi^+, \tag{6.8.20}$$

$$S^+ + n \longrightarrow n + \pi^+, \tag{6.8.21}$$

$$S^+ + n \longrightarrow p + \pi^o, \tag{6.8.22}$$

where S^+ is a spurion with the isospin properties of the π^+. Thus, in analogy with ordinary π N scattering, we see that, if the π N states in in (6.8.20), (6.8.21) and (6.8.22) are in the pure I = 3/2 state, then the ratio of the rates for (6.8.17), (6.8.18) and (6.8.19) should be

$$1 : (1/9) : (2/9), \tag{6.8.23}$$

by Clebsch-Gordan coefficients. Hence the ratio of charged to neutral pion production should be given by

$$\frac{N(\pi^+)}{N(\pi^\circ)} = 5 \; . \qquad (6.8.24)$$

Similarly, if the final πN state is pure $I = \frac{1}{2}$, then the ratio of rates becomes

$$0 : (4/9) : (2/9), \qquad (6.8.25)$$

so that

$$\frac{N(\pi^+)}{N(\pi^\circ)} = 2 \; . \qquad (6.8.26)$$

Preliminary experiment shows that (6.8.24) and (6.8.26) are correct.

CHAPTER SEVEN: THE K^0 AND CP VIOLATION.

7.1 The K^0 Decay Matrix Element.

The K^0 decay

$$K \longrightarrow 2\pi \qquad (7.1.1)$$

has a matrix element of the form

$$M = f_\theta \psi_K \overline{\psi}_{\pi 1} \overline{\psi}_{\pi 2} , \qquad (7.1.2)$$

where f_θ is an unknown constant. From a consideration of dimensions, we may deduce that

$$f_\theta = G(x_\theta m_K)^3 , \qquad (7.1.3)$$

where x_θ is a constant in the order of unity. We find that the rate of the so-called 'theta' decay of the K^0 is given by

$$W_\theta = (f_\theta^2 \sqrt{(m_K^2 - 4m_\pi^2)})/(16\pi m_K^2) . \qquad (7.1.4)$$

Using the experimental lifetime of the theta decay, (1)

$$(0.866 \pm 0.007) \times 10^{-10} , \qquad (7.1.5)$$

we obtain

$$x_\theta \sim 0.7 . \qquad (7.1.6)$$

We note that, whereas the rate of θ decay is (2)

$$(1.128 \pm 0.006) \times 10^{10} \quad s^{-1} , \qquad (7.1.7)$$

the rate of the decay

$$K^+ \longrightarrow 2\pi \qquad (7.1.8)$$

is only

$$(1.707 \pm 0.015) \times 10^6 \quad s^{-1} . \qquad (7.1.9)$$

Thus the $K_{\pi 2}$ decay of the K^+ is about 7000 times less probable than that of the K^0. As we showed in 6.3, the only isospin state available to the pions in (7.1.8) is $I = 2$, and thus (7.1.8) involves

$$\Delta I = (3/2) , \qquad (7.1.10)$$

violating the selection rule (6.7.1). However, since the rate for the decay (7.1.8) is only (7.1.9), we see that the rule (6.7.1) is obeyed to a high degree of accuracy.

In the decays

$$K \longrightarrow \pi^+ + \pi^-, \tag{7.1.11}$$

$$K \longrightarrow \pi^\circ + \pi^\circ, \tag{7.1.12}$$

a final state with I = 1 or I = 2 is forbidden by (6.7.1), so that we are forced to conclude that the final two-pion states of (7.1.11) and (7.1.12) have a total isospin of zero. We describe the isospin wave function of the first pion by the vector $\underline{a}$, and of the second, by $\underline{b}$. In order to obtain a total isospin of zero, we write the final state isospin as the scalar product of the vectors $\underline{a}$ and $\underline{b}$:

$$\underline{a} \cdot \underline{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 . \tag{7.1.13}$$

Taking into account that the π^+ is described by the wave function

$$a_+ = (a_1 + ja_2)/\sqrt{2} , \tag{7.1.14}$$

the π^- by

$$a_- = (a_1 - ja_2)/\sqrt{2} , \tag{7.1.15}$$

and the π° by

$$a_o = a_3 , \tag{7.1.16}$$

we may rewrite the scalar product (7.1.13) in terms of the new variables (7.1.14), (7.1.15) and (7.1.16):

$$\underline{a} \cdot \underline{b} = a_+ b_- + a_- b_+ + a_o b_o . \tag{7.1.17}$$

Since the probability of charged pion formation is proportional to

$$|a_+ b_-|^2 + |a_- b_+|^2 , \tag{7.1.18}$$

while that for neutral pions is proportional to

$$|a_o b_o|^2 , \tag{7.1.19}$$

the rule (6.7.1) gives (see 6.7)

$$\frac{W(K \longrightarrow \pi^+ + \pi^-)}{W(K \longrightarrow \pi^\circ + \pi^\circ)} = 2 , \tag{7.1.20}$$

or

$$B(K) = \frac{W(K \longrightarrow \pi^\circ + \pi^\circ)}{W(K \longrightarrow \pi^\circ + \pi^\circ) + W(K \longrightarrow \pi^+ + \pi^-)} = \frac{1}{3} . \tag{7.1.21}$$

Taking into account a possible small admixture of amplitude with $\Delta I = (3/2)$, we predict

$$B(K) = 0.29 \rightarrow 0.37 . \tag{7.1.22}$$

Experiments give

$$B(K) = (0.3123 \pm 0.0026) , \tag{7.1.23}$$

in good agreement with theory.

In the case of the three-pion decay

$$K \longrightarrow \pi + \pi + \pi^{\circ}, \qquad (7.1.24)$$

matrix element calculations become more complex than in the two-pion case. In analogy with (7.1.2), we write the matrix element for (7.1.24) as

$$M = f_{\tau}\, \Psi_K\, \bar{\Psi}_{\pi 1}\, \bar{\Psi}_{\pi 2}\, \bar{\Psi}_{\pi 3} \quad , \qquad (7.1.25)$$

where f_τ is a dimensionless variable dependent upon the energy of the final state pions in the so-called 'tau' decay (7.1.24). Since the pion energy never exceeds about 25 MeV, it is reasonable to assume that f_τ is roughly constant with energy. Hence, in analogy to (7.1.3), we may write

$$f_\tau \sim G(x_\tau\, m_K)^2 \quad , \qquad (7.1.26)$$

where x_τ is again a constant near unity. We may now obtain an expression for the rate of (7.1.24), and integrating over the momenta of π_2 and π_3, we have

$$W_\tau = (f_\tau^2)/(2\pi)^5 \int (d\underline{p}_1)/(16 m_K m_\pi) \,.\, (4\pi q)/(2m_\pi) \,, \qquad (7.1.27)$$

where q is the absolute value of the 3-momentum of π_2 or π_3 in its c.m.s. Introducing a constant Q, known as the disintegration energy of the decay, defined

$$Q = m_K - 3m_\pi \,, \qquad (7.1.28)$$

we find that

$$W_\tau = (f_\tau^2\, Q^2)/(2^7\, \pi^2\, 3\sqrt{3}\, m_K) \,. \qquad (7.1.29)$$

Writing f_τ in the form (7.1.26), we observe that

$$x \sim 1 \,, \qquad (7.1.30)$$

substituting the experimental value for the rate of the decay (7.1.24) of (3)

$$(6.42 \pm 0.13) \times 10^6 \quad s^{-1} \,. \qquad (7.1.31)$$

We note that in the case of the three-pion decay, the $K^{\pm}_{3\pi}$ decay is not suppressed.

We now perform a similar analysis in terms of isospin on the $K^{\circ}_{3\pi}$ decay as we did on the $K^{\circ}_{2\pi}$ one above. As we found in (6.3.72), the final pion states in the decays

$$K^{+} \longrightarrow \pi^{+} + \pi^{-} + \pi^{+}, \quad (\tau) \qquad (7.1.32)$$

$$K^{+} \longrightarrow \pi^{\circ} + \pi^{\circ} + \pi^{+}, \quad (\tau') \qquad (7.1.33)$$

$$K^{\circ} \longrightarrow \pi^{\circ} + \pi^{\circ} + \pi^{\circ}, \qquad (7.1.34)$$

$$K^{\circ} \longrightarrow \pi^{\circ} + \pi^{+} + \pi^{-}; \qquad (7.1.35)$$

tend to have I = 1. As above, we denote the isospin wave functions of the pions by $\underline{a}$, $\underline{b}$ and $\underline{c}$. Thus the general three-pion state will be described by

$$\underline{A} = \underline{a}\cdot(\underline{b}\cdot\underline{c}) + \underline{b}\cdot(\underline{c}\cdot\underline{a}) + \underline{c}\cdot(\underline{a}\cdot\underline{b}) . \tag{7.1.36}$$

The component

$$A^{+} = a_{+}(\underline{b}\cdot\underline{c}) + b_{+}(\underline{c}\cdot\underline{a}) + c_{+}(\underline{a}\cdot\underline{b}) \tag{7.1.37}$$

corresponds to the decays (7.1.32) and (7.1.33), and the component

$$A^{o} = a_{o}(\underline{b}\cdot\underline{c}) + b_{o}(\underline{c}\cdot\underline{a}) + c_{o}(\underline{a}\cdot\underline{b}) \tag{7.1.38}$$

corresponds to the neutral decays (7.1.34) and (7.1.35). We now write A^{+} and A^{o} in the form

$$A^{+} = a_{+} b_{+} c_{-} + a_{+} b_{-} c_{+} + a_{+} b_{o} c_{o} + a\ b_{+} c_{+} + a_{+} b_{+} c_{-} +$$

$$+ a_{o} b_{+} c_{o} + a_{+} b_{-} c_{+} + a_{-} b_{+} c_{+} + a_{o} b_{o} c_{+} , \tag{7.1.39}$$

$$A^{o} = a_{o} b_{+} c_{-} + a_{o} b_{-} c_{+} + a_{o} b_{o} c_{o} + a_{-} b_{o} c_{+} + a_{+} b_{o} c_{-} +$$

$$+ a_{o} b_{o} c_{o} + a_{+} b_{-} c_{o} + a_{-} b_{+} c_{o} + a_{o} b_{o} c_{o} . \tag{7.1.40}$$

By the rule (6.7.1), we obtain the following relations between the decay modes:

$$\frac{W(K^{+}\longrightarrow 2\pi^{+} + \pi^{-})}{W(K^{+}\longrightarrow 2\pi^{o} + \pi^{+})} = \frac{|2a_{+}b_{+}c_{-}|^{2} + |2b_{+}c_{+}a_{-}|^{2} + |2a_{+}c_{+}b_{-}|^{2}}{|2a_{+}b_{o}c_{o}|^{2} + |2b_{+}c_{o}a_{o}|^{2} + |2a_{o}c_{+}b_{o}|^{2}} = \frac{12}{3} = 4 \tag{7.1.41}$$

$$\frac{W(K^{o} \rightarrow 3\pi^{o})}{W(K^{o}\rightarrow \pi^{+} + \pi^{-} + \pi^{o})} = \frac{|3\ a_{o}\ b_{o}\ c_{o}|^{2}}{|a_{o}b_{+}c_{-}|^{2} + |a_{o}b_{-}c_{+}|^{2} + |b_{o}c_{+}a_{-}|^{2} + |b_{o}c_{-}a_{+}|^{2} + |c_{o}a_{+}b_{-}|^{2} + |c_{o}a_{-}b_{+}|^{2}} = \frac{9}{6} = \frac{3}{2} \tag{7.1.42}$$

Further, since it has been found that 50 % of all K^{o} particles decay into the 3π channel,

$$\frac{W(K^{+}\longrightarrow \pi^{+} + \pi^{+} + \pi^{-}) + W(K^{+}\longrightarrow \pi^{o} + \pi^{o} + \pi^{+})}{W(K^{o}\longrightarrow \pi^{o} + \pi^{o} + \pi^{o}) + W(K^{o}\longrightarrow \pi^{+} + \pi^{-} + \pi^{o})} = 1 . \tag{7.1.43}$$

However, due to the mass difference within the pion triplet, we must make some phase-space corrections to our ratios (7.1.41), (7.1.42), (7.1.43). These become

$$1.24 : 4 \sim 0.32 \tag{7.1.44}$$

$$1.49 : (2 \times 1.23) \sim 1.8 \tag{7.1.45}$$

$$(3 \times 1.49 + 2 \times 1.23) : (4 + 1.26) \sim 1.3 . \tag{7.1.46}$$

The first of these ratios (7.1.44) we obtained before (6.3.71), and we find that our two predictions and the experimental value agree well. However, the experimental ratios for (7.1.45) and (7.1.46) are not yet accurate enough for comparison.

7.2 The Dual Properties of the K^0.

The parity of the θ meson may be determined by knowing the total parity of its two decay pions. Since the pions have odd parity, the total parity of the di-pion system is given by

$$P = (-1)^L , \qquad (7.2.1)$$

where L is the orbital angular momentum of the final state. Thus, assuming the pions to have zero spin, the possible J^P assignments for the θ become

$$J^P = 0^+, 1^-, 2^+, 3^-, \ldots\ldots \qquad (7.2.2)$$

From the decay mode

$$\theta \longrightarrow \pi^0 + \pi^0 , \qquad (7.2.3)$$

it is obvious that the θ is a boson, and hence the permitted spin-parity assignments are reduced to

$$J^P = 0^+, 2^+, 4^+ \ldots\ldots , \qquad (7.2.4)$$

i.e. even spin and even parity. We now attempt to evaluate the spin-parity of the τ meson, with decay mode

$$\tau \longrightarrow 3\pi . \qquad (7.2.5)$$

In order to find the total parity of the three-pion system, we consider it as a di-pion of orbital momentum L, with another pion of orbital momentum M relative to the di-pion. Thus we have, in analogy to (7.2.1),

$$P = (-1)^3 (-1)^L (-1)^M , \qquad (7.2.6)$$

and since symmetry demands even parity for the di-pion, (7.2.6) now becomes

$$P = -(-1)^M . \qquad (7.2.7)$$

We find that the spin of the three-pion system obeys the inequality

$$|M - L| \leqslant J \leqslant |M + L| , \qquad (7.2.8)$$

and thus the first few possible spin-parity assignments are

$$J^P = 0^-, 1^+, 2^-, 2^+ \ldots\ldots \qquad (7.2.9)$$

Hence assuming parity conservation, the lowest allowed J^P for the K^0, if the θ and τ mesons are indeed the same particle, should be 2^+ from (7.2.4) and (7.2.9). However, angular distribution of decay products favour zero spin for the K^0. It was for this reason that, in 1956, Lee and Yang (4) suggested that parity might not be conserved in the weak interaction, thus allowing the K^0 to possess zero spin, and, as yet, undetermined parity. As we saw in 3.7, parity is, in fact, violated by the weak interaction. More sensitive angular distribution experiments (5) have shown that the K^0 has $J^P = 0^-$. A further important feature of the θ and τ mesons is that, due to the difference in phase-space factors for their decays, their lifetimes differ by a factor of over 100:

$$T_\theta = (0.886 \pm 0.007) \times 10^{-10} \text{ s} , \quad (7.2.10)$$

$$T_\tau = (5.179 \pm 0.040) \times 10^{-8} \text{ s} . \quad (7.2.11)$$

According to the formula (5.1.27), the K^0 and $\bar{K}^0$ should have strangenesses of + 1 and -1 respectively. However, since the kaons are the lightest strange particles, they must decay via the strangeness-violating weak interaction. Since the final states from the K^0 and $\bar{K}^0$ decays contain only non-strange particles, it is impossible to ascertain from a study of its decay products whether a particular particle was initially a K^0 or a $\bar{K}^0$. For this reason, Fermi considered that the K^0 and $\bar{K}^0$ were, in fact, indistinguishable. However, whereas K^0 mesons could be produced both in associated production reactions:

$$\pi^- + p \longrightarrow K^0 + \Lambda , \quad (7.2.12)$$

and in charge exchange

$$K^+ + n \longrightarrow K^0 + p , \quad (7.2.13)$$

$\bar{K}^0$ mesons could only be produced by charge exchange

$$K^- + p \longrightarrow \bar{K}^0 + n , \quad (7.2.14)$$

or in pairs with K and K^0:

$$\pi^+ + p \longrightarrow K^+ + \bar{K}^0 + p . \quad (7.2.15)$$

From (7.2.12), (7.2.13), (7.2.14) and (7.2.15) we see that, if strangeness is conserved in the strong interactions (and there evidence to support this view), then more K^0 than $\bar{K}^0$ particles should be produced, implying a distinction between the two entities. The solution to this paradox was put forward by Gell-Mann and Pais (6) in 1955. Since the decay products of the K^0 and $\bar{K}^0$ are identical, we see that the two particles may transform into one another via virtual pion states. These transitions involve $|\Delta S| = 2$, and hence they must be two-stage or second-order weak effects, with a very low amplitude. However, this hypothesis indicates that, if we have a pure K^0 beam at $t = 0$, then at a later time, we shall have a superposition of both K^0

and $\bar{K}^o$. This situation is peculiar to the K^o meson, since it is the only particle which is able to undergo virtual transitions to its antiparticle state. All baryons and leptons may not commute with their antiparticles because of baryon and lepton conservation, the photon is its own antiparticle, the charged pions are forbidden to commute with each other by charge conservation, and the π^o is its own antiparticle. Thus we write the composition of a K^o beam observed at any finite distance from its source as

$$|K(t)\rangle = A(t)|K^o\rangle + B(t)|\bar{K}^o\rangle. \qquad (7.2.16)$$

In order to determine the functions A and B, we must now find what eigenstates of the weak interaction are responsible for K^o decay. At this point, we shall assume invariance under the combined operator CP. For the K^o and $\bar{K}^o$ themselves, we have

$$CP\ |K^o\rangle = -|\bar{K}^o\rangle, \qquad (7.2.17)$$

$$CP\ |\bar{K}^o\rangle = -|K^o\rangle, \qquad (7.2.18)$$

since, assuming the spin of the kaon to be zero, in the rest frame of the K^o, CP will have the same effect as C on its own. The minus signs on the right-hand sides of (7.2.17) and (7.2.18) are purely arbitrary. Thus we see that the K^o and $\bar{K}^o$ are not themselves the required eigenstates of CP. However, writing

$$|K_1^o\rangle = (1/\sqrt{2})\ (|K^o\rangle + |\bar{K}^o\rangle), \qquad (7.2.19)$$

$$|K_2^o\rangle = (1/\sqrt{2})\ (|K^o\rangle - |\bar{K}^o\rangle), \qquad (7.2.20)$$

we find that

$$CP\ |K_1^o\rangle = |K_1^o\rangle, \qquad (7.2.21)$$

$$CP\ |K_2^o\rangle = -|K_2^o\rangle, \qquad (7.2.22)$$

so that K_1^o and K_2^o are eigenstates of CP. In terms of K_1^o and K_2^o, we find that

$$|K^o\rangle = (1/\sqrt{2})\ (|K_1^o\rangle + j|K_2^o\rangle), \qquad (7.2.23)$$

$$|\bar{K}^o\rangle = (1/\sqrt{2})\ (|K_1^o\rangle - j|K_2^o\rangle). \qquad (7.2.24)$$

The fact that the state vector of $\bar{K}^o$ is the complex conjugate of that of K^o is suggested by electric charge continuity equations of the type (1.4.12). We note that the phases of K_1^o and K_2^o are always purely arbitrary, so that we may introduce a factor e^{jG} at will.

We now examine the effect of the operator CP on the final pion states in K^0 decay. We showed above (7.2.4) that the parity of the two pion system was even. For the $\pi^0 \pi^0$ system, it is obvious that C = +1, since the π^0 is its own antiparticle. Strictly, the effect of the P operator on a system containing two particles is to interchange their spatial co-ordinates, so that

$$P\ |\pi^+\pi^-\rangle \quad = \quad |\pi^-\pi^+\rangle \quad . \qquad (7.2.25)$$

If the product of the intrinsic parities[1] of the two pions in (7.2.25) had not been even, then the right-hand side of the equation would have been negative. The C operator transforms each particle into its antiparticle, and thus

$$CP\,|\pi^+\pi^-\rangle \quad = \quad |\pi^+\pi^-\rangle \quad . \qquad (7.2.26)$$

Since the state on the right-hand side of (7.2.26) is identical to the initial state in (7.2.25), we may deduce that the CP parity of the two-pion system is always even. However, the situation becomes more complex when we attempt to evaluate the three-pion CP parity. As above, we write the orbital angular momentum of the di-pion $\pi^+\pi^-$ system as L, and the orbital momentum of the π^0 with respect to the di-pion as M. Thus

$$CP|\pi^+\pi^-\pi^0\rangle \quad = \quad (-1)^3\,(-1)^L\,(-1)^M\ C|\pi^+\pi^-\pi^0\rangle \quad , \qquad (7.2.27)$$

following (7.2.6). Writing the di-pion and the π^0 separately, (7.2.27) becomes

$$-(-1)^{(L+M)}\ C|\pi^+\pi^-\rangle\,|\pi^0\rangle = -(-1)^{(L+M)}\,(-1)^L\,|\pi^+\pi^-\pi^0\rangle$$
$$= \quad -(-1)^M\,|\pi^+\pi^-\pi^0\rangle \quad . \qquad (7.2.28)$$

From (7.2.28) we see that for M = 0, the 3π system has CP = -1. Since the three π^0 mesons in the decay (7.1.34) are identical, Bose symmetry[2] demands that they have M even, so that CP = -1. In the charged pion mode (7.1.35), states with M = 1 are strongly inhibited by angular momentum barrier effects. Thus we are forced to conclude that the 2π mode has CP = +1, while the 3π mode has CP = -1. As we saw above, the K_1^0 has CP = +1, and the K_2^0 has CP = -1. If we are to assume CP invariance, this means that the K_1^0 may only decay into the 2π channel, while the K_2^0 may only decay into the 3π one. Thus, unlike the K^0 and $\bar{K}^0$, the composite states K_1^0 and K_2^0 may be distinguished by their decay modes. As with the Θ and Υ mesons, the different types of decay for the K_1^0 and K_2^0 cause a difference in lifetimes between the two particles.

7.3 Phenomena in K^o Beams.

We consider first the development of a K^o beam with time. The K^o particles produced in a reaction of the type

$$\pi^- + p \longrightarrow \Lambda^o + K^o \qquad (7.3.1)$$

will be 50% K_1^o and 50% K_2^o mesons, immediately after production, before any decays have occured. A xenon bubble chamber has been used to show that

$$0.53 \pm 0.05 \qquad (7.3.2)$$

of all K^o particles decay by the 2π mode. The reason for this is that the K^o particles produced in (7.3.1) will be a superposition of the K_1^o and K_2^o states according to (7.2.23). Since the lifetime of the K_1^o is much shorter than that of the K_2^o, the ratio of K_1^o to K_2^o in a K^o beam will decrease until finally, the beam will be pure K_2^o. We now wish to obtain an expression for the amplitudes of the states K_1^o and K_2^o in a developing K^o beam. We know that when a particle is undergoing exponential decay of the form

$$N(t) = N(0)\ e^{-kt}, \qquad (7.3.3)$$

we must multiply its wave function by a phase—space factor (see Appendix C)

$$e^{-\frac{1}{2}\Gamma t}, \qquad (7.3.4)$$

where

$$\Gamma = 1/T, \qquad (7.3.5)$$

as well as by the standard factor

$$e^{-jmt}, \qquad (7.3.6)$$

where m is the particle mass. Often we write

$$M = m - \tfrac{1}{2} j \Gamma, \qquad (7.3.7)$$

so that the combined phase-space factors (7.3.4) and (7.3.6) become

$$e^{-jMt}. \qquad (7.3.8)$$

Thus, in terms of the factor (7.3.8), we may write the complete K^o wave function as

$$|\psi(t)\rangle = (1/\sqrt{2})(|K_1^o\rangle\ e^{-jM(1)t} + j\,|K_2^o\rangle\ e^{-jM(2)t}), \qquad (7.3.9)$$

where $M(1)$ is the value of M for the K_1^o, and M_2 for the K_2^o. (7.3.9) yields, as expected,

$$|\psi(0)\rangle = (1/\sqrt{2})(|K_1^o\rangle + j\,|K_2^o\rangle), \qquad (7.3.10)$$

in agreement with (7.2.23). We now wish to find the intensity of K_1^o and K_2^o after a given time t. Writing $\psi(t)$ explicitly in terms of K^o and $\bar{K}^o$, we obtain from (7.3.9):

$$\psi(t) = \tfrac{1}{2}(|K^o\rangle + |\bar{K}^o\rangle)\, e^{-jM(1)t} + \tfrac{1}{2}(|K^o\rangle - |\bar{K}^o\rangle)\, e^{-jM(2)t} \qquad (7.3.11)$$

In order to find the K^o intensity, we multiply the wave function (7.3.11) by its complex conjugate, following the Born interpretation (1.4.9), and extract the terms in K^o:

$$N(K^o) \propto \tfrac{1}{4}\,(e^{-\Gamma(1)t} + e^{-\Gamma(2)t} + 2\cos((m_2 - m_1))\, e^{-\frac{1}{2}(\Gamma(1)+\Gamma(2))t}) \qquad (7.3.12)$$

Similarly, for the $\bar{K}^o$, we obtain

$$N(\bar{K}^o) \propto \tfrac{1}{4}\,(e^{-\Gamma(1)t} + e^{-\Gamma(2)t} - 2\cos((m_2 - m_1))\, e^{-\frac{1}{2}(\Gamma(1)+\Gamma(2))t}) \qquad (7.3.12)$$

As expected,

$$N(K^o) + N(\bar{K}^o) \propto \tfrac{1}{2}(e^{-\Gamma(1)t} + e^{-\Gamma(2)t}) \;. \qquad (7.3.13)$$

From times short compared with $T(2) = 1/\Gamma(2)$,

$$N(K^o) \propto \tfrac{1}{4}\,(1 + e^{-\Gamma(1)t} + 2\cos(\Delta mt)\, e^{-\frac{1}{2}\Gamma(1)t}) \;, \qquad (7.3.14)$$

$$N(\bar{K}^o) \propto \tfrac{1}{4}\,(1 + e^{-\Gamma(1)t} - 2\cos(\Delta mt)\, e^{-\frac{1}{2}\Gamma(1)t}) \;, \qquad (7.3.15)$$

where Δm is the $K_1^o - K_2^o$ mass difference, so that the intensities of K^o and $\bar{K}^o$ oscillate with frequency Δm.

We now consider the phenomenon of regeneration, which allows us to obtain a numerical value for Δm. After about a hundred K_1^o lifetimes, our K^o beam will be pure K_2^o , and

$$|\psi\rangle = (1/\sqrt{2})\,(|K^o\rangle + |\bar{K}^o\rangle) \;. \qquad (7.3.16)$$

However, if we direct our K^o beam on to a target, then the strong interactions which take place within the target will alter the phases of the particles, so that (7.3.16) becomes

$$|\psi\rangle = (1/\sqrt{2})\,(a|K^o\rangle + b|\bar{K}^o\rangle) \;. \qquad (7.3.17)$$

Three basic types of strong interaction affect the K^o beam: scattering from single nucleons, scattering from complete nuclei, and coherent scattering from all the nuclei in the target. The latter is known as 'transmission regeneration', since, as we shall see, we have regenerated a number of K_1^o particles, which form a secondary beam parallel to the K_2^o one. Writing (7.3.17) in terms of K_1^o and K_2^o, we have

$$|\psi\rangle = ((a - b)/2)|K_1^o\rangle + ((a + b)/2)|K_2^o\rangle . \qquad (7.3.18)$$

Since K^o and $\bar{K}^o$ undergo different strong interactions within the target,

$$a > b , \qquad (7.3.19)$$

and thus we are forced to conclude that (7.3.18) implies that a number of K_1^o particles have been regenerated. Let f_{21} be the probability that a K_1^o is produced from an incoming K_2^o via the strong interaction.

$$f_{21} = (a - b) . \qquad (7.3.20)$$

We assume that the incoming K_2^o beam may be described by a plane wave of momentum p_2. Let the strong scattering process occur at a distance x from the edge of the regenerator slab, and let it produce a K_1^o beam with momentum p_1. Thus the amplitude for the state K_1^o on the second edge of a slab of thickness L is

$$A_1 = \exp(j\, k_2\, x)\;\; f_{21}\;\; \exp(j\, k_1\, (L - x)) . \qquad (7.3.21)$$

Experimentally, the amplitude is slightly lower than (7.3.21), since some of the K_1^o mesons may already have decayed by the time they emerge from the target, and we are assuming zero decay probability for the K_1^o. Let the rest lifetime of the K_1^o be T_1. By relativity (see Appendix A), we calculate that the K_1^o lifetime observed in the laboratory frame is

$$T = (1/(\sqrt{1 - (p_1/E_1)^2}) = \gamma T_1 . \qquad (7.3.22)$$

Thus we may rewrite the amplitude (7.3.21):

$$A = A_1 \exp(-(E_1/k_1)(L - x)/2\gamma T_1) . \qquad (7.3.23)$$

We now wish to find the energy E_1 of the outgoing K_1^o in terms of the energy E_2 of the incoming K_2^o. We know that

$$k_2 = k_1 + p , \qquad (7.3.24)$$

$$\sqrt{k_2^2 + m_2^2} + M = \sqrt{k_1^2 + m_1^2} + \sqrt{M^2 + p^2} , \qquad (7.3.25)$$

where p is the momentum of the recoiling nucleus in the target and M is its mass. Assuming M to be much greater than any other energy involved, substituting for p in (7.3.25), and solving for k_1, we obtain

$$k_1 - k_2 = (m_2/k_2)\,(m_2 - m_1). \qquad (7.3.26)$$

Denoting the total effective number of nuclei per unit length in the target by N, the amplitude for the K_1^o at the second edge of the target becomes

$$A = \int_0^L N\,dx\ \exp(jk_2x)\, f_{21}\ \exp(jk_1(L - x)\ \exp(-(m_1/k_1)((L - x)/2T_1)))$$

$$= (N f_{21})/(j(k_2 - k_1) + (m_1/k_1)(1/2T_1))\ (\exp(jk_2L) - \exp(jk_1L - (m_1/k_1)(L/2T_1)))\ . \quad (7.3.27)$$

Thus the probability of finding a K_1^o at the second side of the target is given by

$$W(K_1^o) = W_o\,(1 - 2\cos(2\delta g)\, e^{-g} + e^{-2g})\ , \quad (7.3.28)$$

where

$$g = (m_1/2k_1)\ (L/T_1)\ , \quad (7.3.29)$$

$$\delta = (k_1 - k_2)/m_1 \quad k_1 T_1 \approx m\, T_1 \quad (7.3.30)$$

and W_o is the probability of observing a K_1^o at the second side of an infinitely-thick target. Thus oscillations known as 'strangeness oscillations' occur in a regenerated K_1^o beam. The equation (7.3.28), from which we might theoretically calculate Δm, is modified by multiple strong interactions and by the non-forward scattering of some of the main beam particles.

By studying the frequency of strangeness oscillation in regeneration experiments, it is possible to find the magnitude of δ , but not its sign. By this method, it has been deduced that (7)

$$\delta = (0.60 \pm 0.15)\ . \quad (7.3.31)$$

However, it is also possible to measure both the sign and the magnitude of δ in a single experiment. We take as an example of an experiment of this type that of Mehlhop et al. (8) in 1968. A K^+ beam of momentum 0.99 GeV/c was made to impinge upon a copper target in which charge exchange took place, resulting in the production of a K^o beam. At a distance of a few K_1^o lifetimes from this target was placed an iron regenerator slab. The beam of K_1^o emerging from the regenerator consisted of a superposition of K_1^o particles from the original beam and K_1^o particles regenerated from K_2^o's in the target. The total K_1^o intensity, which was dependent upon the magnitudes and phases of the original and regenerated K_1^o wave functions, was measured by means of a number of foil spark chambers. The phase of the original wave is proportional to the original K_1^o momentum p_1 , and the phase of the regenerated wave to the regenerated K_1^o momentum p_R . Due to the mass difference between K_1^o and K_2^o ,

$$p_R \neq p_2 \neq p_1\ . \quad (7.3.31)$$

A number of strong interaction effects in the regenerator affect the phase of the regenerated K_1^0 wave. By measuring the intensity of the K_1^0 beam for differing values of $T = (D + L)/B$, where D is the distance from the copper target to the iron slab and B is the mean free path of the K_1^0,

$$B \sim c\,T \sim 2.66 \text{ cm}, \qquad (7.3.32)$$

interference phenomena showed that

$$(m_{K_2} - m_{K_1}) = (0.44 \pm 0.06)\, \hbar / T_1 \quad s^{-1} \qquad (7.3.33)$$

reverting to S.I. units. A more sensitive measurement of the $K_2^0 - K_1^0$ mass difference has been made (9) by observing interference between the decays

$$K_1^0 \longrightarrow \pi^+ + \pi^-, \qquad (7.3.34)$$

$$K_2^0 \longrightarrow \pi^+ + \pi^-, \qquad (7.3.35)$$

where the K_1^0 particles have been regenerated from K_2^0's. The decay (7.3.35) is an example of CP violation, which will be discussed in the following section. This method yields

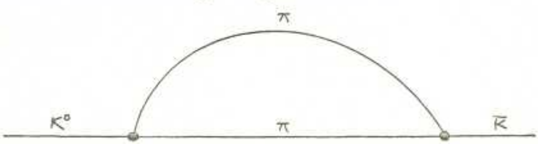
$$\Delta m = (0.480 \pm 0.024)\, \hbar / T_1 \quad s^{-1}. \qquad (7.3.36)$$

Using the best available values for $\hbar$ and T_1, an average of a number of recent experiments gives (10)

$$\Delta m = (5.403 \pm 0.035) \times 10^9 \; \hbar \; s^{-1} \qquad (7.3.37)$$

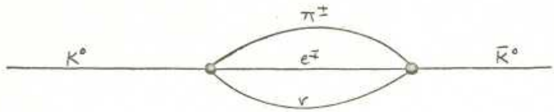
$$= (5.123 \pm 0.033) \times 10^{-25} \; J. \qquad (7.3.38)$$

We now append a brief survey of the theory underlying the $K_2^0 - K_1^0$ mass difference. This is thought to have arisen because of the existence of the so-called 'self-energy' diagrams such as



(7.3.39)

We now consider through which states the $K^0 - \bar{K}^0$ commutation depicted in (7.3.39) may occur, since it is the matrix elements for conversion into these states which determine the magnitude of the K^0 self-energy. Obviously the states available to the K_1^0 must be different from those available to the K_2^0, otherwise no mass difference would result. We assume throughout this discussion absolute CP invariance, although this is not fully justified. However, the contribution made to the mass difference by CP violation is very small, in the order of the CP-violating amplitude, which is about 10^{-3}. First, we consider the possibility that $K^0 - \bar{K}^0$ commutation occurs via semileptonic intermediate states. However, taking the diagram



(7.3.40)

as an example, we see that one vertex must always involve $\Delta Y = -\Delta Q$, violating the selection rule (5.3.16). There is good evidence to show that any violation of (5.3.16) has an amplitude of under 10^{-3}, so that leptonic contributions to Δm will be negligible. Thus the main contribution to the K_1^0 and K_2^0 self-energies appears to come from commutation via the 2π state and/or via the π^0 and η^0 (3π) poles. From CP invariance, we see that only the s wave ($J = 0$) 2π state contributes to the K_1^0, while the π^0 and η^0 poles and the p wave ($J = 1$) 2π state contribute to the K_2^0 self-energy.

Because of the rule (6.7.1), only the $I = 0$ s wave 2π state is important. We introduce the Lorentz invariant self-energy operator:

$$\Pi(m_K^2) = 2W\frac{(2\pi)^3}{2} j \int d^4x \left\langle K_1^0 \right| T_{if} \left| K_1^0 \right\rangle , \qquad (7.3.41)$$

where T_{if} is the T matrix element (see 2.7) for the $K^0 - \bar{K}^0$ transition, and W is the energy of the K_1^0. We find that, since

$$W \approx m_K \qquad (7.3.42)$$

in the rest frame of the K_1^0,

$$(\Delta E)_{K_1^0} = -(1/2m_{K^0}) \quad \mathrm{Re}\, \Pi(m_K^2) , \qquad (7.3.43)$$

$$1/T_1 = \Gamma_1 = (2/2m_{K^0}) \quad \mathrm{Im}\, \Pi(m_K^2) . \qquad (7.3.44)$$

Calculating $\Pi(W^2)$, we may deduce that (7.3.43) is primarily dependent upon the effective mass of the 2π system, G. If

$$2m_\pi < G < m_{K^0} , \qquad (7.3.45)$$

then the 2π state gives a positive contribution to the K_1^0 self-energy, and hence a negative one to Δm. If

$$G > m_{K^o} \quad , \tag{7.3.46}$$

then 2π makes a positive contribution to Δm. Thus, if an s wave resonance[3] were to exist with a mass near to that of the K^o, and with the 2π decay mode dominant, then its mass would determine the magnitude and sign of the 2π contribution to Δm. However, experiments show that no such resonance exists, unless we are to identify our resonance with the unconfirmed pole $\epsilon(600)$. Thus the 2π state probably does not make an important contribution to Δm. The π^o, η^o and ρ^o (p wave 2π) states contribute only to the K_2^o self-energy. The π^o and η^o contributions are given by

$$(\Delta m)_{\pi,\eta} = (\Delta E)_{K_2^o} = (1/2m_K)(((|a_{K_2^o\pi^\bullet}|^2)/(m_K^2 - m_\pi^2)) - $$
$$- ((|a_{K_2^o\eta^\bullet}|^2)/(m_\eta^2 - m_K^2) , \tag{7.3.47}$$

where $a_{K_2^o\pi^\bullet}$ is proportional to the amplitude of the decay

$$K^o \longrightarrow \pi^o + \pi^o \quad , \tag{7.3.48}$$

and similarly $a_{K_2^o\eta^\bullet}$ is proportional to the amplitude of

$$K^o \longrightarrow \eta^o \longrightarrow \pi^+ + \pi^- + \pi^o \quad . \tag{7.3.49}$$

However, it is not usually possible to calculate the amplitudes for the decays (7.3.48) and (7.3.49), even in terms of the whole K_2^o decay rate. SU(3) (see chapter 8) does make this possible, but the predictions of exact SU(3) are contrary to the experimental value of Δm, and the degree of SU(3) violation is not, at present, known. Contributions to Δm may also come from the vector mesons $\rho(770)$ and $\omega(783)$, and from the axial vector mesons A_1 (1100) and $\omega'(1675)$. However, once again, the amplitudes a are not known, and so no calculation of Δm is possible.

The experimental value of Δm is perhaps the best evidence against $|\Delta Y| = 2$ transitions. For if these were allowed, then $K^o - \bar{K}^o$ commutation could occur without an intermediate state of zero hypercharge, for example

$$K^o \longrightarrow \bar{\Lambda} + N \longrightarrow \Lambda + \bar{N} \longrightarrow \bar{K}^o \quad . \tag{7.3.50}$$

In (7.3.50), both the first and last transitions would occur by the strong interaction, while the middle one would still take place via the weak interaction. However, since (7.3.50) is a first-order weak interaction, we find that its contribution to Δm is much greater than that of, for example, (7.3.39). Summing over all reactions of the type (7.3.50), we find that

$$\Delta m \sim (G\, m_N^2)/(G\, m_N^2)^2 \quad (1/T_1) \sim 10^5\,(1/T_1)\,, \tag{7.3.51}$$

where G is the weak coupling constant. If only $|\Delta Y| = 1$ transitions are allowed, then

$$\Delta m \sim (1/T_1)\,. \tag{7.3.52}$$

(7.3.52) is in near agreement with experiment (7.3.38), but (7.3.51) is in violent disagreement. This means that we may set an upper limit on the $|\Delta Y| \geqslant 2$ amplitude of 10^{-5}.

7.4 CP Violation.

In 1964, Christenson, Cronin, Fitch and Turlay (11), while studying regeneration phenomena, detected the decay

$$K_2^0 \longrightarrow 2\pi\ , \tag{7.4.1}$$

showing that CP was violated. A target was placed at 30° to a 30-GeV proton beam. Gamma rays from this target were attenuated by placing a 4-cm-thick lead block behind it, and charged particles were removed from the secondary beam by means of an electromagnet. The beam was then collimated, and 18 m further on, a second lead collimator led it into a helium-filled bag. Decay products from here were detected by means of two spectrometers placed symmetrically 22° from the main beam. Each of these spectrometers consisted of a pair of spark chambers separated by a magnet and triggered by scintillation counters and a water Cerenkov detector. The spark chmabers were triggered if and only if a main beam particle decayed into charged particles with velocities greater than about 0.75 c. Decays of the type (7.4.1) were detected in the following manner. When two particles of opposite electric charge were detected in coincidence by the spark chambers, the momentum and effective mass, on the assumption that the two particles were pions, was calculated. The effective or invariant mass was found from the formula

$$M_{eff} = c^{-2}((E_1 + E_2)^2 + c^2(\underline{p}_1 + \underline{p}_2)^2)^{\frac{1}{2}}\ , \tag{7.4.2}$$

using

$$E_i = (c^2\, p_i^2 + M^2\, c^4)^{\frac{1}{2}}\,. \tag{7.4.3}$$

(7.4.2) corresponded to the rest mass of the decaying particle if and only if the particle decayed by the mode (7.4.1). For this decay,

$$M_{eff} = M_K \sim 493 \text{ MeV}/c^2 , \tag{7.4.4}$$

but for the normal decay

$$K_2^0 \longrightarrow \pi^+ + \pi^- + \pi^0 , \tag{7.4.5}$$

since only the charged pions are observed,

$$280 \text{ MeV}/c^2 < M_{eff} < 363 \text{ MeV}/c^2. \tag{7.4.6}$$

For

$$K_2^0 \longrightarrow \pi + \mu + \nu , \tag{7.4.7}$$

$$280 \text{ MeV}/c^2 < M_{eff} < 516 \text{ MeV}/c^2 , \tag{7.4.8}$$

and for

$$K_2^0 \longrightarrow \pi + e + \nu , \tag{7.4.9}$$

$$280 \text{ MeV}/c^2 < M_{eff} < 536 \text{ MeV}/c^2 . \tag{7.4.10}$$

For the reactions (7.4.7) and (7.4.9), M_{eff} would vary smoothly, and would not be peaked around 493 MeV/c^2, as for the modes (7.4.1) and (7.4.5). In a two-body decay, the sum of the three-momenta of the decay products and the initial direction of the decaying particle should be the same, but for three-body decays, the two vectors are usually at an angle to each other. By both angular and effective mass measurements, it was found that 45 ± 9 out of 22 700 K_2^0 particles decayed via the 2π mode. This number was at least an order of magnitude too large to be explained by regeneration of K_1^0's in the helium or elsewhere. Christenson et al. showed that

$$R = \frac{W(K_2^0 \longrightarrow \pi^+ + \pi^-)}{W(K_2^0 \rightarrow \text{all charged modes})} = (2.0 \pm 0.4) \times 10^{-3}, \tag{7.4.11}$$

and

$$|\eta_{+-}| = W(K_2^0 \longrightarrow \pi^+ + \pi^-)/W(K_1^0 \longrightarrow \pi^+ + \pi^-) = (1.90 \pm 0.05) \times 10^{-3} . \tag{7.4.12}$$

CP violation has also been observed in the decay

$$K_2^0 \longrightarrow \pi^0 + \pi^0 . \tag{7.4.13}$$

One technique (12) used to detect (7.4.13) was to observe gamma rays produced by the decaying π^0 mesons by means of metal plates, in which the 'pair production' reaction

$$\gamma \longrightarrow e^+ + e^- \tag{7.4.14}$$

took place. One difficulty encountered was to correct for decays of the type

$$K_2^0 \longrightarrow 3\pi^0 \longrightarrow 6\gamma , \qquad (7.4.15)$$

when two of the final gamma rays did not materialize, simulating a $2\pi^0$ decay. This correction was made by means of the 'Monte Carlo' computer calculation, in which decaysof the type (7.4.16) were tested to find out how often they would simulate (7.4.13) decays. Another method used to observe (7.4.13) was (13) to measure the energies of the final gamma rays from π^0 decay. Only in the $2\pi^0$ decay will a gamma ray have an energy of above 170 MeV in the c.m.s. system of the K^0. The γ-ray energies were found by a spark-chamber magnetic spectrometer, but transformation to the K^0 c.m.s. demanded a knowledge of the K^0 momentum. This was obtained by regulating the beam in short bursts, and making time-of-flight velocity measurements. Knowing the kaon mass, the momentum could thus be calculated. The rate for 2π decay as a fraction of 3π decay was deduced by measuring the number of gamma rays with energies above and below 170 MeV. Correcting for processes other than (7.4.15) which could produce low-energy gamma rays, the result

$$|\eta_{oo}| = \frac{W(K_2^0 \longrightarrow 2\pi^0)}{W(K_1^0 \longrightarrow 2\pi^0)} = (2.9 \pm 0.5) \times 10^{-3} \qquad (7.4.16)$$

was obtained. The currently acknowledged values of the K_2^0 decay CP violation parameters are (14)

$$|\eta_{+-}| = (2.17 \pm 0.07) \times 10^{-3} , \qquad (7.4.17)$$

$$|\eta_{oo}| = (2.25 \pm 0.09) \times 10^{-3} . \qquad (7.4.18)$$

We write the total weak Hamiltonian as

$$H'_W = H_W + H_- , \qquad (7.4.19)$$

where H_W is the usual weak Hamiltonian, which we assume obeys $|\Delta Y| = 1$, and H_- is our new CP violating Hamiltonian. The final pions in the decay (7.4.1), since they have zero total angular momentum, must be in an $I = 0$ or an $I = 2$ state. We define the quantities:

$$\epsilon = (\langle I=0 | H'_W | K_2^0 \rangle)/(\langle I=0 | H'_W | K_1^0 \rangle) , \qquad (7.4.20)$$

$$\epsilon' = (\langle I=2 | H'_W | K_2^0 \rangle)/(\langle I=2 | H'_W | K_1^0 \rangle) , \qquad (7.4.21)$$

$$\omega = (\langle I=2 | H'_W | K_1^0 \rangle)/(\langle I=0 | H'_W | K_1^0 \rangle) , \qquad (7.4.22)$$

$$\eta_{+-} = |\eta_{+-}| e^{j\theta_{+-}} = (a_2(+-))/(a_1(+-)) , \qquad (7.4.23)$$

$$\eta_{oo} = |\eta_{oo}| e^{j\theta_{oo}} = (a_2(oo))/(a_1(oo)) , \qquad (7.4.24)$$

$$\rho = (a_1(+-))/(a_1(oo)) , \qquad (7.4.25)$$

where $|\eta_{+-}|$ and $|\eta_{oo}|$ were defined in (7.4.12) and (7.4.16) respectively.

We now write (7.2.19) as

$$|K^o_S\rangle = (1/\sqrt{2}) \; (p|K^o\rangle + q|\bar{K}^o\rangle) , \qquad (7.4.26)$$

and (7.2.20):

$$|K^o_L\rangle = (1/\sqrt{2}) \; (p|K^o\rangle - q|\bar{K}^o\rangle) , \qquad (7.4.27)$$

where K^o_S and K^o_L are defined

$$K^o_S \longrightarrow 2\pi , \qquad (7.4.28)$$

$$K^o_L \longrightarrow 3\pi . \qquad (7.4.29)$$

If there were no CP violation, then

$$K^o_1 \equiv K^o_S , \qquad K^o_2 \equiv K^o_L . \qquad (7.4.30)$$

In terms of the quantaties p and q in (7.4.26) and (7.4.27), we find that

$$\epsilon = (p - q)/(p + q) . \qquad (7.4.31)$$

A similar relation holds for ϵ'. We now see that ϵ and ϵ' are measures of CP violation, since if there exists perfect CP invariance,

$$p = q = 1 , \qquad (7.4.32)$$

so that (7.4.26) becomes (7.2.19), and

$$\epsilon = 0 . \qquad (7.4.33)$$

If both CP and CPT invariance hold, then we also have

$$\epsilon' = 0 . \qquad (7.4.34)$$

However, in the event of CP violation, only one of the relations (7.4.33) and (7.4.34) must be true. We see that ω(7.4.22) is a measure of the validity of the $|\Delta I| = \frac{1}{2}$ rule (6.7.1), since, if the rule is satisfied, no transitions to an I = 2 final state are allowed. Experiments indicate (15) that

$$|\omega| \sim 1/20 . \qquad (7.4.35)$$

Assuming $|\omega|^2$ to be near zero, we find that

$$\eta_{+-} \approx \epsilon + (1/\sqrt{2}) \epsilon' , \qquad (7.4.36)$$

$$\eta_{oo} \approx \epsilon - \sqrt{2} \epsilon' , \qquad (7.4.37)$$

so that

$$|\epsilon| \leqslant (2/3)|\eta_{+-}| + (1/3)|\eta_{oo}| , \qquad (7.4.38)$$

$$|\epsilon'| \leqslant (\sqrt{2}/3)(|\eta_{+-}| + |\eta_{00}|) \ . \qquad (7.4.39)$$

Using the experimental values (7.4.17) and (7.4.18), we obtain

$$|\epsilon| \leqslant 3 \times 10^{-3} \ , \qquad (7.4.40)$$

$$|\epsilon'| \leqslant 2.9 \times 10^{-3} \ . \qquad (7.4.41)$$

We may show that

$$\langle K_1 \mid K_2 \rangle = (|p|^2 - |q|^2)/(|p|^2 + |q|^2) \leqslant 2|\epsilon| \ . \qquad (7.4.42)$$

Thus the states K_1 and K_2 are nearly orthogonal, i.e. they have only a small overlap. From our definition of K_1^0 and of K_2^0, we see that the value of (7.4.42) is a measure of the CP violation in the $K^0 - \bar{K}^0$ system, and it would evidently be zero if there were no CP violation. In (7.4.23) and (7.4.24), we wrote η_{+-} and η_{00} as the product of a magnitude and a phase. The magnitudes have been found by measuring the ratio of CP-violating to CP-conserving decays (7.4.17), (7.4.18), and the phases by measuring interference between the decays (7.3.34) and (7.3.35). The time-dependent interference term is proportional to

$$(\Delta m t - \theta_{+-}) \ , \qquad (7.4.43)$$

and thus the determination of θ_{+-} is sensitive to the value of Δm. Using the value (7.3.38), experiments give (16)

$$\theta_{+-} = (46 \pm 15)^{\circ} \ . \qquad (7.4.44)$$

Studies of interference decay product angular distribution yield (17)

$$\theta_{+-} = (46.6 \pm 2.5)^{\circ} \ . \qquad (7.4.45)$$

The value of Re ϵ may be found from measurements of charge asymmetry in the reactions

$$K_L^0 \longrightarrow \pi^{\pm} + l^{\mp} + \nu_l \ , \qquad (7.4.46)$$

giving (18)

$$\mathrm{Re}\,\epsilon = (1.09 \pm 0.18) \times 10^{-3} \ . \qquad (7.4.47)$$

Assuming

$$|\eta_{00}| = |\eta_{+-}| \ , \qquad (7.4.48)$$

we find that

$$\theta_{\epsilon} = (42.7 \pm 1.3)^{\circ} \ , \qquad (7.4.49)$$

$$\theta_{00} = (49 \pm 13)^{\circ} \ , \qquad (7.4.50)$$

and we see that the value of $|\epsilon'|$ must be small compared with $|\epsilon|$.

CPT invariance implies the precise equality of the total rate for

$$K^{+} \longrightarrow 3\pi \qquad (7.4.51)$$

and for

$$K^{-} \longrightarrow 3\pi \ , \qquad (7.4.52)$$

or

$$\Gamma(+ + -) + \Gamma(0\,0\,+) = \Gamma(- - +) + \Gamma(0\,0\,-) \ . \qquad (7.4.53)$$

However, the Dalitz plots for the processes (7.4.51) and (7.4.52) are not congruent, because of the existence of a final-state strong interaction between the pions. Thus, different rates for the $(+ + -)$ mode of the K and for the $(- - +)$ mode of the K^- decay would constitute evidence for CP but not necessarily for CPT violation. However, simple consideration of the symmetric isospin state (6.3.66) for the final pions also yields an equality of the partial rates[4] for (7.4.51) and (7.4.52), so that this is not a sensitive test of CP invariance. Experimentally (19),

$$\frac{W(K \longrightarrow \pi^+ + \pi^+ + \pi^-)}{W(K \longrightarrow \pi^- + \pi^- + \pi^+)} = 1.0004 \pm 0.002 \ , \qquad (7.4.54)$$

which is consistent with no deviation between the partial rates for (7.4.51) and (7.4.52). A more satisfactory test of CP invariance in the decays

$$K^{\pm} \longrightarrow 3\pi \qquad (7.4.55)$$

is afforded by measuring the final-state energy spectrum. If CP is conserved, then this should be identical for the K^+ and the K^-. Experimental measurement of the slopes in the spectra for the decays (7.4.55) yield (20)

$$S^{+}(+ + -) = 0.11 \pm 0.015 \ , \qquad (7.4.56)$$

$$S^{-}(- - +) = 0.115 \pm 0.02 \ , \qquad (7.4.57)$$

which is consistent with CP invariance.

If CP were exactly conserved, then the decay

$$K_1^{0} \longrightarrow \pi^+ + \pi^- + \pi^0 \ , \qquad (7.4.58)$$

although not forbidden (see 7.2), would be inhibited by an angular momentum barrier factor of order

$$(Q/m_K)^2 \sim 1/200 \ , \qquad (7.4.59)$$

Q being the disintegration energy of the decay (7.4.58). Such a low rate is effectively unobservable because of the large background of

$$K_2^{0} \longrightarrow 3\pi \qquad (7.4.60)$$

decays. However, the decay

$$K_1^0 \longrightarrow 3\pi^0 \qquad (7.4.61)$$

is forbidden by CP invariance, since the π^0's in the final state are identical particles. Writing

$$(A_1(+ - 0))/(A_2(+ - 0)) = x(+ - 0) + j\, y(+ - 0) , \qquad (7.4.62)$$

experiments show that (21)

$$x(+ - 0) = 0.14 \pm 0.32 , \qquad (7.4.63)$$

$$y(+ - 0) = 0.33 \pm 0.61 , \qquad (7.4.64)$$

excluding any CP-violating amplitude with greater strength than the CP-conserving one in (7.4.61). Further experiments (22) yield

$$|\eta_{000}|^2 \leqslant 1.2 . \qquad (7.4.65)$$

Alternatively, we might reveal CP violation in charge asymmetry in the decay

$$K_L^0 \longrightarrow \pi^+ + \pi^- + \pi^0 . \qquad (7.4.66)$$

However, experiments (23) show that

$$A = N(T_{\pi^+} > T_{\pi^-})/N(T_{\pi^+} < T_{\pi^-}) = (0 \pm 5)\,\% , \qquad (7.4.67)$$

which is consistent with CP invariance.

Finally, we consider CP violation in the semileptonic decays of the K^0. Assuming that $\Delta Y/\Delta Q = 1$ or -1, we have four decays to discuss:

$$K^0 \longrightarrow \pi^- + l^+ + \nu_l , \qquad (7.4.68)$$

$$\bar{K}^0 \longrightarrow \pi^+ + l^- + \bar{\nu}_l , \qquad (7.4.69)$$

$$K^0 \longrightarrow \pi^+ + l^- + \bar{\nu}_l , \qquad (7.4.70)$$

$$\bar{K}^0 \longrightarrow \pi^- + l^+ + \nu_l . \qquad (7.4.71)$$

As we saw above, CP violation in hadronic decays does not necessarily involve

$$\epsilon \neq 0 ; \qquad (7.4.72)$$

but the semileptonic decays, since they have only $I = 0$ in the final state, do demand $\epsilon \neq 0$ for CP violation, making them of special interest. Neglecting the final state interaction between the $\pi^\pm$ and $l^\mp$, CPT invariance implies congruent Dalitz plots and equal and opposite $\mu^\pm$ polarization in K_{l3}^0 and $\bar{K}_{l3}^0$ decays. In most experiments, K^0 and $\bar{K}^0$ beams are allowed to propagate in vacuo until the short-lived component K_S^0 has completely died out through 2π decay, so that only K_L^0's remain, whose semileptonic decays (7.4.68), (7.4.69), (7.4.70) and (7.4.71) may then be studied. CP violation in these decays will cause a slight departure from the CP-invariant forms of

Dalitz plot and muon polarization. Writing

$$X = (A(\Delta Y/\Delta Q = -1))/(A(\Delta Y/\Delta Q = +1)) , \tag{7.4.73}$$

CP invariance implies

$$\frac{W(l^+)}{W(l^-)} \approx 1 + 4 \,\mathrm{Re}\, \epsilon \left[\frac{1 - |X|^2}{|1 + X|^2}\right] , \tag{7.4.74}$$

neglecting higher powers of ϵ . Thus, by studying charge asymmetry, we may find a value for ϵ so long as we know X from other sources. However, even if $X = 0$, the semileptonic K^0 decays should exhibit charge asymmetry if $\epsilon \neq 0$. The parameter

$$\delta_L(l) = \frac{W(l^+) - W(l^-)}{W(l^+) + W(l^-)} \tag{7.4.75}$$

has been measured experimentally as (24)

$$\delta_L(e) = (2.24 \pm 0.36) \times 10^{-3} , \tag{7.4.76}$$

$$\delta_L(\mu) = (4.05 \pm 1.7\,) \times 10^{-3} , \tag{7.4.77}$$

yielding

$$\frac{W(e^+)}{W(e^-)} = (1.0043 \pm 0.0007) , \tag{7.4.78}$$

$$\frac{W(\mu^+)}{W(\mu^-)} = (1.0081 \pm 0.0027) . \tag{7.4.79}$$

Statistics show that the departure from unity in (7.4.78) and (7.4.79) is significant, but that the inequality of electron and muon values is not significant, so that electron-muon universality is upheld. An average of the values (7.4.76), (7.4.77) gives

$$\delta_L = 2 \,\mathrm{Re}\, \epsilon \left[\frac{1 - |X|^2}{|1 + X|^2}\right] = (2.32 \pm 0.35) \times 10^{-3} . \tag{7.4.80}$$

However, we see that we cannot determine X and ϵ separately from (7.4.80), and thus we need an independent value of X. Assuming $X = 0$, we have

$$\mathrm{Re}\, \epsilon = (1.16 \pm 0.18) \times 10^{-3} , \tag{7.4.81}$$

but Re ϵ is very sensitive to the value of X, and measurements on decays such as

$$\Sigma^+ \longrightarrow n + l^+ + \nu_l \tag{7.4.82}$$

have only set an upper limit of 0.1 on $|X|^2$. Thus we must determine the magnitude of the complete factor

$$\frac{1 - |X|^2}{|1 + X|^2} \qquad (7.4.83)$$

by experiment. Measurements of the relative intensity of the regenerated K_S^o component in a K^o beam give the value of (7.4.83) as (25)

$$(1.06 \pm 0.06)\ , \qquad (7.4.84)$$

and thus we obtain

$$\mathrm{Re}\ \epsilon = (1.09 \pm 0.18) \times 10^{-3}, \qquad (7.4.85)$$

which is close to the value (7.4.81) for which we assumed $X = 0$. Thus we have established some degree of CP violation in the K^o semileptonic decays.

7.5 Models for CP Violation.

When CP violation was first detected in the decay (7.4.1) a number of theories were advanced to account for this effect. One was that the Bose symmetry used to calculate the CP parity of the two-pion system was incorrect, but this was invalidated by the observation of the decay (7.4.13). Another suggestion was that the decay

$$K_2 \longrightarrow S + K_1 \longrightarrow S + \pi^+ + \pi^- \qquad (7.5.1)$$

took place, where S is a particle with CP = -1 and with a mass less than the $K_L^o - K_S^o$ mass difference. However, if this were the situation, then no interference between the K_2 final state $\pi^+\pi^- S$ and the K_1 final state $\pi^+\pi^-$ would be expected, but this definitely occurs. A further explanation was that the effect was due to a long-range 'galactic' interaction (26) which coupled with different strength to matter and antimatter and hence to the K^o and $\bar{K}^o$. Thus, in a region in which matter exists in greater quantities than antimatter, the 'galactic' interaction would cause the K_L^o and K_S^o to be a mixture of CP eigenstates. However, this theory predicts the rate for the decay (7.4.1) to be proportional to

$$\gamma^{2J}\ , \qquad (7.5.2)$$

where γ is the Lorentz factor (see appendix A) and J is the spin of the quantum or propagator (see chapter 9) of the new field. Experiments on the reaction (7.4.1) for varying K_2^o momenta have shown that there is no observable velocity-dependence for the reaction rate.

We write the total Hamiltonian of the strong, electromagnetic and weak interactions as

$$H = H_+ \quad H_- \,, \tag{7.5.3}$$

where

$$CP\, H_\pm\, (CP)^{-1} = \pm H_\pm \,. \tag{7.5.4}$$

From the existence of the decay (7.4.1) it is obvious that an interaction with CP = -1 does exist, but its properties are almost unknown. The CP-violating reactions may be classified according to their hypercharge selection rules. We first consider the case in which H_- is a $\Delta Y = 2$ operator. We write

$$H = H_0 + H_W \,, \tag{7.5.5}$$

where H_0 is the Hamiltonian for the strong and electromagnetic interactions, and H_W is the normal CP-conserving weak Hamiltonian obeying the selection rule

$$|\Delta Y| = 1 \,. \tag{7.5.6}$$

Since all the weak hadronic decays obey (7.5.6), H_- cannot be the Hamiltonian responsible for them. However, H_- has its effect by giving nonequal off-diagonal terms to the mass matrix[5], causing $p \neq q$ in (7.4.26), and hence a nonvanishing amplitude for the decay (7.4.1). In order to account for the observed branching ratios, we find that the contribution to the $\Delta Y = 2$ amplitude made by H_- must be about $|\eta_{+-}|$ times that from second-order $\Delta Y = 1$ CP-conserving effects. Thus H_- describes an interaction with coupling constant

$$\sim 10^{-9}\, (G\, m_N^2/4\pi) \,, \tag{7.5.7}$$

where G is the usual weak coupling constant:

$$G\, m_N^2 \sim 10^{-5} \,. \tag{7.5.8}$$

Because of its small coupling constant (7.5.7), the interaction H_- ($\Delta Y = 2$) is known as the superweak interaction. It was first postulated by Wolfstein in 1964 (27). Neglecting terms of order 10^{-9}, we find that there will be no other CP-violating weak effects except for those associated with the K^0, since it is for this state only that terms of order

$$(Fm_N^2/4\pi)(Gm_N^2/4\pi)^2 \sim 10^{-3} \tag{7.5.9}$$

appear through the existence of the mass matrix[6]. The superweak interaction model assumes that the normal weak Hamiltonian is T invariant, so that the amplitudes a_i appearing in the definitions (7.4.23), (7.4.24) and (7.4.25) must all be real. We now introduce the definitions

$$\epsilon' = ((p - q(a_2/a_2))/(p + q))(a_2/a_0)\, e^{jx} , \tag{7.5.10}$$

$$\omega = ((p + q(a_2/a_2))/(p + q))(a_2/a_0)\, e^{jx} , \tag{7.5.11}$$

where a_r now represents the amplitude to the state with $I = r$ (previously, it denoted the amplitude for the K_r^o decay into a particular channel). The reality of amplitudes thus implies

$$\epsilon' = \epsilon\omega , \tag{7.5.12}$$

$$\eta_{+-} = \epsilon = \eta_{oo} . \tag{7.5.13}$$

From the so-called 'unitarity condition'[7],

$$((\Delta m\, T_S\, 2\, \mathrm{Re} + \mathrm{Im}\,(\epsilon'^{*}\omega)) \simeq \mathrm{Re}\,\epsilon\, \tan\Theta_\epsilon , \tag{7.5.14}$$

we obtain

$$(2\Delta m)T_S\, \mathrm{Re}\,\epsilon = \mathrm{Re}\,\epsilon \tan\Theta_\epsilon (1 + |\omega|^2) , \tag{7.5.15}$$

making use of (7.5.12) and (7.5.13). Neglecting ω^2, (7.5.15) yields

$$\tan\Theta_\epsilon = 2\Delta m\, T_S , \tag{7.5.16}$$

$$\Theta_\epsilon = (42.7 \pm 1.3)^o . \tag{7.5.17}$$

Thus, from (7.5.13) we predict

$$\Theta_{+-} = \Theta_{oo} = (42.7 \pm 1.3)^o , \tag{7.5.18}$$

which is correct within experimental error (7.4.44), (7.4.45), (7.4.50). However, the prediction

$$\eta_{+-} = \epsilon \tag{7.5.19}$$

does not agree with experiment (7.4.17), (7.4.81). The final prediction

$$\eta_{oo} = \epsilon \tag{7.5.20}$$

has not yet been tested, due to lack of satisfactorily-accurate experiments.

The second case which we consider is that when H_- is a $\Delta Y = 1$ operator. Here H_- has the same hypercharge selection rule as the normal weak Hamiltonian H_W , so that most weak interactions should have CP-violating amplitudes. Taking the example (7.4.1), we find that

$$F/G \sim 10^{-3} , \tag{7.5.21}$$

where F is the coupling constant of the CP-violating Hamiltonian H_-. However, as we have mentioned above, experiments have failed to reveal definite CP violation in any other weak processes, although the level of accuracy is rarely 10^{-3}. Thus, until further experiments have been performed we have no method of deciding between the weak and superweak interaction theories. We now discuss two particular models for the $\Delta Y = 1$ CP-violating Hamiltonian H_- . The first postulates that CP violation occurs in the $\Delta I = 3/2$ part of the weak hadronic Hamiltonian (28). If it were definitely established that

$$|\eta_{oo}| \neq |\eta_{+-}| , \qquad (7.5.22)$$

then this would favour the $\Delta I = 3/2$ model, since ϵ' could no longer be small compared to ϵ, and there would have to be CP violation in the $\Delta I = 3/2$ component of the hadronic Hamiltonian. However, if (7.5.22) is not true, i.e.

$$|\eta_{oo}| = |\eta_{+-}| , \qquad (7.5.23)$$

then CP violation must occur exclusively in the $\Delta I = \frac{1}{2}$ part of the Hamiltonian. Assuming that the CP-violating interaction satisfies an exact $\Delta I = 3/2$ selection rule, while the CP-conserving one satisfies exactly $\Delta I = \frac{1}{2}$, we find that

$$(p^2/q^2) = ((1 - \epsilon)/(1 + \epsilon))^2 , \qquad (7.5.24)$$

which may be shown to imply that neither the K^o nor the $\bar{K}^o$ receive self-energy contributions from the 2π channel. From (7.5.24) it seems likely that will be very small, so that the decay (7.4.1) must occur primarily because

$$\mathrm{Im}\,(a_2/a_o) \neq 0 \qquad (7.5.25)$$

(isospin definition), indicating that

$$|\epsilon'| \gg |\epsilon| . \qquad (7.5.26)$$

(7.5.26) is not favoured by current experimental evidence, since

$$|\eta_{+-}| = \left| \epsilon + \frac{1}{\sqrt{2}}\epsilon' \right| \sim 1.89 \times 10^{-3} , \qquad (7.5.27)$$

and

$$\mathrm{Re}\,\epsilon \sim 1.1 \times 10^{-3} , \qquad (7.5.28)$$

so that

$$\epsilon' < \epsilon . \qquad (7.5.29)$$

Thus we are forced to conclude that the $\Delta I = 3/2$ model is probably not correct.

The second model for the Hamiltonian H_- ($\Delta Y = 1$) which we consider is known as the 'semiweak' model (29). This is based on the observation that the characteristic amplitude for a first-order CP-conserving weak process is in the order of

$$(Gm_N^2/4\pi) \sim 10^{-6}, \qquad (7.5.30)$$

whereas that for a CP-violating one is

$$\sim (Gm_N^2/4\pi)^{3/2} \sim 10^{-9}. \qquad (7.5.31)$$

We then postulate that both CP-conserving and CP-violating hadronic processes are due to a fundamental semiweak CP-violating interaction with coupling constant

$$f \sim (Gm_N^2/4\pi)^{\frac{1}{2}} \sim 10^{-3}. \qquad (7.5.32)$$

By a sutiable choice of conditions, we ensure that all first-order matrix elements vanish, that all CP-conserving normal weak processes have second-order matrix elements, and that the third-order matrix elements describe CP-violating processes. We choose the semiweak Hamiltonian:

$$H_{SW} = f(\partial/\partial x_r)\, N_r \,, \qquad (7.5.33)$$

where N_r is a neutral current with $\Delta Y = 0$ and $|\Delta Y| = 1$, but with no $\Delta Y = 2$ components. We find that N_r can be so constructed that the first-order matrix elements vanish because of momentum conservation. However, in the second order, we predict $|\Delta Y| = 2$, which is inconsistent with experiment. This problem is overcome by postulating that the $\Delta Y = 2$ part vanishes, which is found to be equivalent to demanding that a particular commutator vanishes. The third-order terms give correct predictions for CP-violating reactions. The 'semiweak' hypothesis may provide the basis for a unified theory of the weak interaction.

In the third case, H_- obeys $\Delta Y = 0$ (it contains no leptonic component) and violates C and T but conserves P, and thus violates CP. Here, CP violation occurs as a result of the cross-term $H_W H_-$ appearing in the second-order matrix elements. This means that in both strong and weak processes (including electromagnetic interactions), there must exist a C and CP-violating amplitude which is about $|\eta_{+-}| \sim 2 \times 10^{-3}$ times as large as the C and CP-conserving one. Thus we may test this model by searching for C and CP (or T) violation in strong and electromagnetic interactions. Since the amplitude for the process (7.4.1) is of the same magnitude as the coupling constant of the electromagnetic interaction, it has been suggested (30) that CP violation might be caused by the electromagnetic interaction. The ratio

$$|\eta_{00}/\eta_{+-}| \tag{7.5.34}$$

is consistent with models in which CP violation in weak processes is due to CP violation in the electromagnetic interaction, since the final two-pion state in the decay (7.4.1) may be any admixture of I = 0 and I = 2 states. To the weak interaction, models with differing values of ΔY probably appear identical, but in the $\Delta Y = 0$ case, we should expect sizeable C- and T-violating amplitudes in the electromagnetic interaction. We split the total hadronic electromagnetic current into two components:

$$J_r^{em} = j_r^{em} + K_r^{em} , \tag{7.5.35}$$

where

$$C\, j_r^{em}\, C^{-1} = -j_r^{em} , \tag{7.5.36}$$

$$C\, K_r\, C^{-1} = K_r . \tag{7.5.37}$$

Thus K is the C-violating amplitude. As usual, the total current must be conserved:

$$(\partial/\partial x_r)\, J_r^{em} = 0 . \tag{7.5.38}$$

We now define two types of charge:

$$Q_J = -j \int j_4^{em}(\underline{x}, t)\, d^3x , \tag{7.5.39}$$

$$Q_K = -j \int K_4(\underline{x}, t)\, d^3x . \tag{7.5.40}$$

The total charge of a system is given by

$$Q_{tot} = Q_J + Q_K . \tag{7.5.41}$$

Obviously, for any known particle

$$Q_K\,|\text{Particle}\rangle = 0 , \tag{7.5.42}$$

since the signs of the charges on all known particles are reversed by the operator C. It is usually acknowledged that (7.5.42) is true for all particles although there is little justification for this assumption.

In order to test the hypothesis outlined above, we must search for C and T (or CP) violation in electromagnetic processes. The reaction

$$\eta^{\circ} \longrightarrow \pi^{+} + \pi^{-} + \pi^{0} \tag{7.5.43}$$

must proceed electromagnetically, since it violates G parity (see 5.3). If (7.5.43) is C-invariant, then the parameter

$$A = (N(E_{\pi^+} > E_{\pi^-}) - N(E_{\pi^-} > E_{\pi^+}))/(N(E_{\pi^+} > E_{\pi^-}) \quad N(E_{\pi^-} > E_{\pi^+}) \tag{7.5.44}$$

will be zero. A number of experiments have been carried out for the purpose of evaluating A, of which we discuss two. In 1966 Larribe et al. (31) obtained η particles by the interaction of 0.82 GeV/c π^+ mesons with liquid deuterium in a bubble chamber according to the reaction

$$\pi^+ + d \longrightarrow p + \eta^0 + p \quad . \tag{7.5.45}$$

Eta decays were identified by a short proton recoil, and particle energies were found by kinematic fitting. In all, 21 000 events were measured, of which 765 fitted the reaction (7.5.45). This experiment yielded

$$A \quad = \quad -0.048 \pm 0.036 \quad , \tag{7.5.46}$$

which is consistent with C invariance. A much larger number of events may be measured if spark chambers are used instead of bubble chambers, and an experiment using spark chambers was performed by Cnops et al. (32). Here, η° particles were produced in a liquid hydrogen target by incident $\bar{}$ mesons with momenta of 0.713 GeV/c according to

$$\pi^- + p \longrightarrow n + \eta \, . \tag{7.5.47}$$

The neutron momentum was measured by time-of-flight analysis in order to find the precise η^0 energy. After a specified time interval, two spark chambers were triggered if and only if they received two oppositely-charged particles, corresponding to the charged pions of eta decay. In order to avoid errors caused by the asymmetry of the magnetic field used to separate the decay pions, this was reversed half-way through the experiment. By the end of the experiment, 10 665 events fitting the reaction (7.5.47) had been studied, yielding

$$A \quad = \quad (0.3 \pm 1.1) \, \% \, . \tag{7.5.48}$$

Parity conservation may easily be checked for the electromagnetic interaction by attempting to observe nuclear transitions which violate parity. By this method, the upper limit for electromagnetic P violation has been set at 10^{-3}. P invariance in the strong interaction has been investigated in great detail by studying nuclear decays. In 1971, Krane et al. (33) polarized hafnium-180 and observed spatial asymmetry in its decay gamma rays, demonstrating that a small component of the strong interaction violates parity. Time reversal invariance in the strong interaction has been verified by measuring the rates for the reactions (34)

$$p + Al^{27} \longrightarrow \alpha + Mg^{24} , \qquad (7.5.49)$$

and these have been shown to be equal to an accuracy of better than 0.3 %. The best test of electromagnetic T invariance is the measurement of a possible electric dipole moment (EDM) of the neutron. The Hamiltonian for electromagnetic interactions between the electric- and magnetic-dipole moments of the neutron may be written

$$H_I = \rho_m \underline{\sigma}.\underline{H} \qquad \rho_e \underline{\sigma}.\underline{E} , \qquad (7.5.50)$$

where ρ_m and ρ_e are the magnitudes of the magnetic- and electric-dipole moments, $\underline{\sigma}$ is the spin vector and $\underline{H}$ and $\underline{E}$ are the magnetic and electric field vectors. It is obvious that $\underline{H}$ and $\underline{\sigma}$ are even under P and odd under T , while $\underline{E}$ is odd under P and even under T. Thus the contribution to the Hamiltonian due to the magnetic-dipole moment is invariant under both P and T, while that due to the electric-dipole moment changes sign under both of these operations. Hence a nonzero neutron EDM would imply P or T violation in the electromagnetic interaction. Writing ($\hbar = c = 1$)

$$EDM = e.f.10^{-5}/M \sim 10^{-19} f \text{ (e cm)}, \qquad (7.5.51)$$

where e is the electronic charge and f is the T or P-violating amplitude, we may measure the neutron EDM. In the experiment of Dress et al. (35), thermal neutrons from a reactor were 'cooled' by passage through a narrow tube of polished nickel with radius of curvature 1 m. Since the critical angle for neutron total internal reflection is inversely proportional to velocity, only low-energy neutrons were transmitted through the tube to strike a magnetized cobalt-iron mirror at grazing incidence. The neutron beam was thus 70 % spin-polarized transversely to its direction of propagation. Having traversed a spectrometer, the neutrons impringed upon an analysing magnet similar to the polarizer, and were reflected to a neutron-sensitive scintillation counter. The transmitted intensity is obviously greatest for those neutrons which do not suffer depolarization in the spectrometer. The spectrometer consisted firstly of a 10 G uniform magnetic field which caused the neutrons to precess with the Larmor frequency (36)

$$v_L = \mu H/h, \qquad (7.5.52)$$

where μ is the neutron magnetic moment and H is the strength of the external magnetic field. Secondly, an RF field with frequency v was applied to the neutron beam, so that at resonance,

$$v = v_L \sim 25 \text{ kHz} , \qquad (7.5.53)$$

the neutron beam was partially depolarized, changing the transmitted intensity, I. Finally, a reversible electric field E of 100 kV/cm was applied in the same direction as the constant magnetic field. The experiment consisted of observing the change in I when E was reversed. If the neutron possessed an EDM in the same direction as its spin, then E would produce an additional small precession for constant v, thus changing I. No effect of this type was observed, so that

$$\text{EDM}_{\text{neutron}} < 3 \times 10^{-22} \text{ e cm} . \qquad (7.5.54)$$

Comparing (7.5.54) with (7.5.51), we see that the experimental value for the neutron EDM sets an upper limit of 10^{-2} on the T-violating amplitude in the electromagnetic interaction. Thus it seems likely that CP violation in the weak interaction is not caused by C or T violation in another known interaction.

CHAPTER EIGHT: THE WEAK INTERACTION AND SU(3).

8.1 The Group SU(3).

SU(3) is an infinite group consisting of all unitary and unimodular 3 X 3 matrices, such that

$$M M^{+} = I , \qquad (8.1.1)$$

$$\det M = 1 . \qquad (8.1.2)$$

A useful method for studying groups is to employ Lie algebra (1). By definition, any matrix M belonging to a matrix group must possess an inverse[1].
Thus there exists a matrix A such that

$$M = e^{A} = I + A + (A^2/2!) + (A^3/3!) \;\ldots.., \qquad (8.1.3)$$

and hence A is the logarithm of M. The set of all matrices whose exponentials also belong to the complete group G are said to constitute the Lie algebra of G. For SU(3), we write

$$M = e^{jh} . \qquad (8.1.4)$$

The unitarity condition (8.1.1) now yields

$$M M^{+} = e^{jh}\, e^{-jh} = I , \qquad (8.1.5)$$

whence

$$M M^{+} = I = M^{+} M . \qquad (8.1.6)$$

Thus M and M^{+} commute; it follows that their logarithms also commute, so that

$$e^{jh}\, e^{-jh^{+}} = e^{j(h-h^{+})} = e^{0} = 1 , \qquad (8.1.7)$$

and hence

$$h = h^{+} , \qquad (8.1.8)$$

i.e. h is Hermitean. Thus we deduce that the Lie algebra of SU(3) consists of 3 X 3 Hermitean matrices. Since these matrices must have three elements on their leading diagonals, they must also have zero trace. Obviously any member h of the Lie algebra of SU(3) may be expressed

$$h = p_1\, g_1 + p_2\, g_2 + p_3\, g_3 + \ldots.. \; p_i\, g_i , \qquad (8.1.9)$$

where p_i are real parameters and g_i are the generators of the Lie group. i is the number of degrees of freedom of any matrix h. All 3 X 3 complex matrices initially depend upon 18 real parameters. The condition (8.1.8) provides three real and three complex relations between the matrix elements, and the tracelessness condition yields one further relation. Thus we find that

$$i = 8 . \tag{8.1.10}$$

One possible choice for the basis or generators of the Lie algebra of SU(3) is (2):

$$g_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad g_2 = \begin{bmatrix} 0 & -j & 0 \\ j & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$g_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad g_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$g_5 = \begin{bmatrix} 0 & 0 & -j \\ 0 & 0 & 0 \\ j & 0 & 0 \end{bmatrix} \qquad g_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \tag{8.1.11}$$

$$g_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -j \\ 0 & j & 0 \end{bmatrix} \qquad g_8 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

From (8.1.3), we see that

$$M = \lim_{n \to \infty} (I + A/n)^n , \tag{8.1.13}$$

and hence

$$A = \lim_{n \to \infty} n \, (M^{(1/n)} - I) . \tag{8.1.14}$$

It may be shown that, for large n, the matrix

$$I + A/n \tag{8.1.15}$$

is an operator of the group G. It is known as an infinitesimal operator. Similarly, as we shall see, the matrices $\frac{1}{2}g_i$ form an explicit representation of the infinitesimal generators of the group SU(3), which we denote by F_i.

We now consider the commutation relations between the SU(3) generators F_i. For this purpose, it suffices to evaluate all commutators of the matrices g_i, and thus we obtain

$$[g_i , g_j] = \sum_{k=1}^{8} 2j\, f_{ijk}\, g_k , \qquad (8.1.16)$$

where f_{ijk} are the so-called 'antisymmetric structure constants' of SU(3):

ijk	f_{ijk}	ijk	f_{ijk}
123	1	345	$\frac{1}{2}$
147	$\frac{1}{2}$	367	$-\frac{1}{2}$
156	$-\frac{1}{2}$	458	$\sqrt{3}/2$
246	$\frac{1}{2}$	678	$\sqrt{3}/2$.
257	$\frac{1}{2}$		

Accordingly, the commutation relations for the generators are

$$[F_i , F_j] = \sum_{k=1}^{8} f_{ijk}\, F_k , \qquad (8.1.17)$$

which are the standard relations for the infinitesimal generators of a group.

Within a given representation of SU(3), it is possible to specify a particular state by giving the eigenvalues of this state under two of the generators of SU(3). Since

$$[F_3 , F_8] = 0 , \qquad (8.1.18)$$

the eigenvalues of a state under these two generators will always be simultaneously measurable. We immediately notice that g_3 is simply the third Pauli spin matrix (5.1.4) bordered with zeroes in order to make it a 3 X 3 matrix. Thus the eigenvalue of a particle state under F_3 is simply the I_3 assignment of that state. Furthermore, we find that F_8 is the hypercharge operator Y. However, F_8 also commutes with F_1 and F_2, so that we may, in fact, diagonalize and hence measure I^2 the total isospin operator,

$$I^2 = I_1^2 + I_2^2 + I_3^2 , \qquad (8.1.19)$$

as well as Y and I_3, at the same time. At this point, we note that, defining the electric charge operator Q in a similar manner as we did in 5.1,

$$Q = F_1 , \qquad (8.1.20)$$

the Gell-Mann - Nishijima - Nakano relation (5.3.22) is verified. Since there exist a number of irreducible representations of the group SU(3) in Hilbert or n-dimensional space, it is necessary to assign a further quantum number to each particle state in order to describe it unambiguously. Initially, two numbers are needed to label each SU(3) representation. One is given by

$$F^2 = \sum_{i=1}^{8} F_i^2 , \qquad (8.1.21)$$

and the other, G, by a complicated third-order polynomial in F_i. The formula for the number of states in an arbitrary irreducible representation[3] is

$$d(F, G) = \tfrac{1}{2}(F + 1)(G + 1)(F + G + 2) . \qquad (8.1.22)$$

d(F, G) is often known as the dimensionality of a particular representation. We now append a table listing the simpler representations of SU(3).

(F, G)	d(F, G)	Name
(0, 0)	1	singlet
(1, 0)	3	triplet
(0, 1)	3^*	triplet
(2, 0)	6	sextet
(0, 2)	6^*	sextet
(1, 1)	8	octet
(3, 0)	10	decuplet
(0, 3)	10^*	decuplet
(2, 2)	27	27 - plet

The $\{3^*\}$ representation is obtained by complex conjugation of the $\{3\}$ representation. The quantum numbers of the singlet representation $\{1\}$ must obviously all be zero, or

$$I = I_3 = Y = 0 . \qquad (8.1.23)$$

Since the group SU(3) was defined in terms of 3 X 3 matrices, there must exist a three-dimensional representation of this group, along with further representations in higher dimensions. Thus $\{3\}$ is the smallest non-trivial representation of SU(3). In order to find the values of Y and I_3 for the three states u_1 , u_2 and u_3 in $\{3\}$, we must solve the two eigenvalue equations:

$$F_3 u_i = I_{3(i)} u_i , \qquad (8.1.24)$$

$$F_8 u_i = Y_i u_i , \qquad (8.1.25)$$

where u_i are the three unit 3-vectors. Multiplying the hypercharge operator F_8 by $\sqrt{3}$, we obtain

	I_3	Y
u_1	$\frac{1}{2}$	1/3
u_2	$-\frac{1}{2}$	1/3
u_3	0	-2/3

(8.1.26)

Thus the representation $\{3\}$ contains an isospinor with $Y = 1/3$ and an isovector with $Y = -2/3$. Similarly, we find that the charges of u_1, u_2 and u_3 are 2/3, -1/3 and -1/3 e respectively. In 1964 Zweig and Gell-Mann postulated that the states in the representation $\{3\}$ might, in fact, have physical significance. They suggested that there exist so-called 'quarks' with fractional charges, which combine together to form the observed hadrons. We shall discuss the quarks in greater detail in the following sections. Finally, we note that there is also another representation, $\{3^*\}$, whose infinitesimal generators are obtained from F_i by complex conjugation.

8.2 The Octet.

On multiplying the representation $\{3^*\}$ by $\{3\}$, we obtain

$$\{3\} \otimes \{3^*\} = \{8\} \oplus \{1\} , \qquad (8.2.1)$$

so that we have a trivial singlet and a new irreducible octet representation of SU(3). Obviously, all particles in this octet must have zero baryon number, and hence they are identified with the mesons. We let P^a_b be the field operator representing the octet of spinless or pseudoscalar mesons, where the upper index a denotes the column within the 3 X 3 matrix concerned, and b the row. Thus we have

$$|\pi^+\rangle = P^2_1 |0\rangle = -|\underline{8};\ 0, 1, +1\rangle \qquad (8.2.2)$$

$$|\pi^-\rangle = P^1_2 |0\rangle = |\underline{8};\ 0, 1, -1\rangle \qquad (8.2.3)$$

$$|\pi^0\rangle = (1/\sqrt{2})(P_1^1 - P_2^2)\ |0\rangle = |\underline{8};\ 0, 1, \ 0\rangle \qquad (8.2.4)$$

$$|K^+\rangle = P_1^3\,|0\rangle = |\underline{8};\ 1, \tfrac{1}{2}, +\tfrac{1}{2}\rangle \qquad (8.2.5)$$

$$|K^0\rangle = P_2^3\,|0\rangle = |\underline{8};\ 1, \tfrac{1}{2}, -\tfrac{1}{2}\rangle \qquad (8.2.6)$$

$$|\bar{K}^0\rangle = P_3^2\,|0\rangle = |\underline{8};\ -1, \tfrac{1}{2}, +\tfrac{1}{2}\rangle \qquad (8.2.7)$$

$$|K^-\rangle = P_3^1\,|0\rangle = -|\underline{8};\ -1, \tfrac{1}{2}, -\tfrac{1}{2}\rangle \qquad (8.2.8)$$

$$|\eta^0\rangle = (-3/\sqrt{6})\,P_3^3\,|0\rangle = |\underline{8};\ 0, 0, \ 0\rangle \qquad (8.2.9)$$

The minus sign appears in front of some of the above states because we have adopted the phase convention

$$I_\pm\ |I, I_3\rangle = \sqrt{(I \mp I_3)(I \pm I_3 + 1)}\ |I, I_3 \pm 1\rangle . \qquad (8.2.10)$$

Since in field theory it is customary to talk of the destruction operator of the $\bar{K}^0$ rather than of the creation operator of the K^0, we now rewrite P_b^a as a destruction operator in matrix form:

$$P_b^a = \begin{bmatrix} (1/\sqrt{6})\,\eta^0 + (1/\sqrt{2})\,\pi^0 & \pi^+ & K^+ \\ \pi^- & (1/\sqrt{6})\,\eta^0 - (1/\sqrt{2})\,\pi^0 & K^0 \\ K^- & \bar{K}^0 & (-2/\sqrt{6})\,\eta^0 \end{bmatrix} \qquad (8.2.11)$$

denoting the field operators of particles by their symbols. Alternatively we may employ the so-called 'octet' notation, so that the particles in the pseudoscalar meson octet have wave functions which may be expressed in terms of ϕ_i ($i = 1, 2, 3, \ldots\ldots 8$). The conversion between the matrix and octet notations may be achieved by means of the formulae

$$P_b^a(x) = (1/\sqrt{2}) \sum_{i=1}^{8} (g_i)_{ab}\ \phi_i(x)\,, \qquad (8.2.12)$$

$$\phi_i(x) = (1/\sqrt{2}) \sum_{a,b=1}^{3} (g_i)_{ba}\ P_b^a(x)\,, \qquad (8.2.13)$$

where g_i are the Gell-Mann matrices (8.1.11). Thus, explicitly, we may write

$$\pi^{\pm}(x) = (1/\sqrt{2})\,(\phi_1(x) \mp j\phi_2(x)), \tag{8.2.14}$$

$$\pi^{0}(x) = \phi_3(x), \tag{8.2.15}$$

$$K^{\pm}(x) = (1/\sqrt{2})\,(\phi_4(x) \mp j\phi_5(x)), \tag{8.2.16}$$

$$K^{0}(x) = (1/\sqrt{2})\,(\phi_6(x) - j\phi_7(x)), \tag{8.2.17}$$

$$\bar{K}^{0}(x) = (1/\sqrt{2})\,(\phi_6(x) + j\phi_7(x)), \tag{8.2.18}$$

$$\eta^{0}(x) = \phi_8(x). \tag{8.2.19}$$

We may obtain another octet by the group multiplication

$$\{3\} \otimes \{3\} \otimes \{3\} = \{10\} \oplus \{8\} \oplus \{8\} \oplus \{1\}. \tag{8.2.20}$$

It is usually assumed that the fundamental states in $\{3\}$ have

$$B = 1/3, \tag{8.2.21}$$

and hence all the particles in the octets of (8.2.20) must be baryons. The first of these octets is usually taken to contain the $J^P = \frac{1}{2}^+$ baryons, so that its field operator in matrix form becomes

$$B^a_b = \begin{bmatrix} (1/\sqrt{6})\Lambda^0 + (1/\sqrt{2})\Sigma^0 & \Sigma^+ & p \\ \Sigma^- & (1/\sqrt{6})\Lambda^0 - (1/\sqrt{2})\Sigma^0 & n \\ \Xi^- & \Xi^0 & (-2/\sqrt{6})\Lambda^0 \end{bmatrix} \tag{8.2.22}$$

The corresponding operator $\bar{B}^a_b$ is given by

$$\bar{B}^a_b = (B^b_a)^+ \gamma_4, \tag{8.2.23}$$

so that

$$\bar{B}^a_b = \begin{bmatrix} (1/\sqrt{6})\bar{\Lambda}^0 + (1/\sqrt{2})\bar{\Sigma}^0 & \bar{\Sigma}^- & \bar{\Xi}^- \\ & (1/\sqrt{6})\bar{\Lambda}^0 - (1/\sqrt{2})\bar{\Sigma}^0 & \bar{\Xi}^0 \\ \bar{p} & \bar{n} & (-2/\sqrt{6})\bar{\Lambda}^0 \end{bmatrix} \tag{8.2.24}$$

The octet notation for baryons may be written down in complete analogy with that for mesons:

$$\Sigma^{\pm}(x) = (1/\sqrt{2})\,(\psi_1(x) \mp j\psi_2(x)), \tag{8.2.25}$$

$$\Sigma^{0}(x) \quad = \quad \psi_3(x) , \qquad (8.2.26)$$

$$p(x) \quad = \quad (1/\sqrt{2})\,(\psi_4(x) - j\psi_5(x)) , \qquad (8.2.27)$$

$$n(x) \quad = \quad (1/\sqrt{2})\,(\psi_6(x) - j\psi_7(x)) , \qquad (8.2.28)$$

$$\Xi^{-}(x) \quad = \quad (1/\sqrt{2})\,(\psi_4(x) + j\psi_5(x)) , \qquad (8.2.29)$$

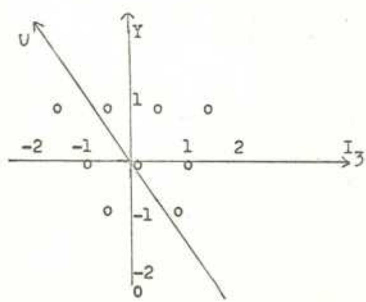
$$\Xi^{0}(x) \quad = \quad (1/\sqrt{2})\,(\psi_6(x) + j\psi_7(x)) , \qquad (8.2.30)$$

$$\Lambda(x) \quad = \quad \psi_8(x) . \qquad (8.2.31)$$

We now consider one of the most important predictions of SU(3) symmetry: the mass formulae. We know that the mass-splitting within isotopic multiplets is caused by electromagnetic self-energy effects arising from the differing values of I_3 within the multiplet. Similarly, the somewhat larger mass splitting within the SU(3) supermultiplets (octets and so on) was attributed to the so-called 'medium strong' interaction (3). Although there is little experimental evidence in favour of a physical interpretation of the medium strong interaction, Ne'eman (4) has suggested that its propagator (see chapter 9) might be the ϕ (1020). In (8.2.20) we saw that there must also exist a decuplet representation of SU(3) $\{10\}$. Its isospin and hypercharge content is given by

$$\{10\} : (I, Y) \quad = \quad (3/2, 1), (1, 0), (\tfrac{1}{2}, -1), (0, -2). \qquad (8.2.32)$$

Upon graphing Y against I_3 , we find that the decuplet forms a triangle. If we draw an axis at 60° to the I_3 axis, we create an axis of unitary or U-spin:



(8.2.33)

We now make the assumption that the strong interaction is scalar in both

I- and U-spin, but that the electromagnetic interaction has a scalar dependence upon U and a vector one upon I_3 . Thus

$$m(I, I_3) = m_o(I) + x(I) I_3 \qquad (8.2.34)$$

gives the charge-splitting within a particular isotopic multiplet. Similarly, we assume that the mass-splitting between different isotopic multiplets is scalar in I and vector in U, so that

$$m(U, U_3) = m_o(U) + y(U) U_3 \ . \qquad (8.2.35)$$

Since

$$U_3 = Y - \tfrac{1}{2} Q \ , \qquad (8.2.36)$$

and since Q is constant within any U-spin (or unitary) multiplet, (8.2.35) becomes

$$m(U, U_3) = m_o(U) + Y \ . \qquad (8.2.37)$$

In the decuplet (8.2.33), no two different particles occupy the same position, and hence the masses of particles in the same unitary multiplet within the decuplet should be linearly related to their values of Y. In this way, we predict the so-called 'equal-spacing' rule (5). The particles in the decuplet are usually identified as follows: the quadruplet consists of the $\Delta(1232)$ πN resonance, the triplet of the Σ resonance $Y^*(1383)$, and the doublet of the $\Xi^*(1531.8)$ resonance. When SU(3) symmetry was first postulated, there existed no particle corresponding to the Y = -2 singlet. It was predicted that a particle, which was named the Ω^-, should occupy this position, and its mass was tentatively calculated by means of the equal-spacing rule

$$m_\Omega - m_{\Xi^*} = m_{\Xi^*} - m_{Y^*} = m_{Y^*} - m_\Delta \qquad (8.2.38)$$

as about 1675 MeV/c^2. As we saw in 6.4, the Ω^- was indeed discovered, and its experimental mass assignment (6.4.16) is in good agreement with that predicted from (8.2.38).

The derivation of a mass formula for the octet is slightly more complicated than for the decuplet, owing to the fact that there exist two superposed particle states with

$$I_3 = Y = 0 \ . \qquad (8.2.39)$$

These states are easily distinguished by isospin as an isosinglet (Λ^0) and an isotriplet (Σ^0). However, they behave identically with respect to

U-spin, so that the $U_3 = 0$ member of the $U = 1$ triplet will be a mixture of the Σ^0 and Λ states. We now introduce the operators U_+ and U_- in analogy to I_+ and I_- (5.1.10), (5.1.11), so that

$$U_- n = a \Lambda + b \Sigma^0 . \qquad (8.2.40)$$

From angular momentum (see Appendix B) we know that

$$U_- |U, U_3\rangle = \sqrt{(U(U+1) - U_3(U_3 - 1))}\ |U, (U_3 - 1)\rangle, \qquad (8.2.41)$$

whence

$$U_- |1, 1\rangle = \sqrt{2}\ |1, 0\rangle . \qquad (8.2.42)$$

Combining (8.2.40) and (8.2.42),

$$U_- |n\rangle = 2 (a|\Sigma^0\rangle + |b \Lambda\rangle) . \qquad (8.2.43)$$

We now transform to the Σ^+ state by applying the operator I_+ :

$$I_+ U_- |n\rangle = \sqrt{2a}\ \sqrt{(I(I+1) - I_3(I_3 + 1))} , \qquad (8.2.44)$$

whence

$$I_+ U_- |n\rangle = \sqrt{2a}\ |\Sigma^+\rangle, \qquad (8.2.45)$$

where the term in Λ^0 has vanished. We may also reach the Σ^+ via the proton, and since I_+ and U_- must commute:

$$U_- I_+ |n\rangle = U_- |p\rangle = |\Sigma^+\rangle . \qquad (8.2.46)$$

Thus

$$a = \tfrac{1}{2} . \qquad (8.2.47)$$

Normalization obviously demands that

$$|a|^2 + |b|^2 = 1 , \qquad (8.2.48)$$

so that

$$b = \sqrt{\tfrac{3}{4}} . \qquad (8.2.49)$$

We arbitrarily choose b to be positive, yielding

$$|U = 1, U_3 = 0\rangle = \tfrac{1}{2}|\Sigma^0\rangle + \sqrt{\tfrac{3}{4}}|\Lambda\rangle, \qquad (8.2.50)$$

so that we have proved the mixing coefficients in (8.2.22). Substituting in (8.2.35) and squaring all coefficients to obtain expectation values, we may write down the Gell-Mann - Okubo formula (6)

$$(m_n + m_{\Xi^0})/2 = (m_{\Sigma^0} + 3m_{\Lambda^0})/4 . \qquad (8.2.51)$$

Experimentally,

$$(m_n + m_{\Xi^0})/2 = 1127.2 \pm 0.4 \ \ MeV/c^2 , \qquad (8.2.52)$$

$$(m_{\Sigma^0} + 3m_\Lambda)/4 = 1134.8 \pm 0.1 \ \mathrm{MeV}/c^2 , \qquad (8.2.53)$$

in near agreement with the prediction (8.2.51). Both (8.2.38) and (8.2.51) are, in fact, special cases of the more general mass formula obtained from perturbation theory:

$$m = a + bY + c(I(I+1) - \tfrac{1}{4}Y^2) , \qquad (8.2.54)$$

where a, b and c are constants depending upon the supermultiplet in question. Since the Klein-Gordan equation (2.2.1) for bosons contains m^2, whereas the Dirac equation only involves m, it seems reasonable to postulate that any SU(3) mass formulae for mesons should contain only the squares of the meson masses. On this hypothesis, we predict

$$m_K^2 = \tfrac{1}{4} m_\pi^2 + \tfrac{3}{4} m_\eta^2 , \qquad (8.2.55)$$

in good agreement with experiment. Deviations from the formula (8.2.55) in higher-mass meson octets are caused by the mixing of octet and singlet states, a strong interaction effect. SU(3) makes a number of useful predictions concerning magnetic moments, and these are also borne out by experiment.

8.3 Applications of SU(3) to the Structure of the Weak Interaction.

From 5.3 we recall that the internal symmetry quantum numbers (Q, Y, I) for the $\Delta Y = 0$ hadron currents are identical to those of the $\pi^\pm$, and that those for the $\Delta Y = 1$ currents are the same as those of the $K^\pm$. Since we know that the $\pi^\pm$ and $K^\pm$ are in the same SU(3) octet, we now postulate that all vector (axial vector) hadron currents also belong to the same vector (axial vector) octet of currents. We write the total semileptonic weak Hamiltonian in the standard form

$$H^S = -(G/\sqrt{2})\,(J_r \bar{L}_r + \text{Herm. conj.}) , \qquad (8.3.1)$$

$$J_r = V_r + A_r , \qquad (8.3.2)$$

where

$$V_r = a V_r^0 + b V_r^1 \qquad (8.3.3)$$

$$= a(V_{1r} + j V_{2r}) + b(V_{4r} + j V_{5r}) , \qquad (8.3.4)$$

$$A_r = a' A_r^0 + b' A_r^1 \qquad (8.3.5)$$

$$= a'(A_{1r} + j A_{2r}) + b'(A_{4r} + j A_{5r}) , \qquad (8.3.6)$$

adopting the convention that the number in the suffixes corresponds to the position of the currents in the current octet, in analogy to (8.2.2) et seq. The current $\bar{V}_r$, for example, may now evidently be written

$$\bar{V}_r = a(V_{1r} - j\,V_{2r}) + b(V_{4r} - j\,V_{5r}) \, . \qquad (8.3.7)$$

In the same notation, the neutral, hypercharge-conserving $\triangle I = 0, 1$ electromagnetic current becomes

$$J_r^{el} = V_{3r} + (1/\sqrt{3})\,V_{8r} \, . \qquad (8.3.8)$$

As a natural extension of the CVC hypothesis for $\triangle Y = 0$ currents discussed in 5.2, we now make the assumption that J^{el}, V^0 and V^1 all belong to the same octet. The remaining components of the vector octet,

$$V_{6r} + j\,V_{7r} \, , \qquad (8.3.9)$$

with internal quantum numbers $Q = 0$, $Y = 1$, $I = \frac{1}{2}$, do not appear to play any important rôle in semileptonic weak processes. Similarly, in the axial vector octet, neither $A_{6r} + j\,A_{7r}$ nor $A_{3r} + n\,A_{8r}$ appear to be significant. In general, the axial vector currents induce transitions between different members of the baryon and meson octets and the vacuum (as in the decay (6.3.4)). At this point, we note that we have assumed that all the axial vector currents transform as members of an octet and not of some higher representation of SU(3). From the baryon fields ψ_i, we may construct an octet of axial vector currents:

$$A_{ir} = -j\,F\,f_{ijk}\,(\psi_j\,j\,\gamma_r\gamma_5\psi_k) + D\,d_{ijk}\,(\psi_j\,j\,\gamma_r\gamma_5\psi_k) \, , \qquad (8.3.10)$$

in 'octet' notation, where d_{ijk} are the symmetric constants of SU(3):

ijk	d_{ijk}
118	1/ 3
146	$\frac{1}{2}$
157	$\frac{1}{2}$
228	1/ 3
247	$-\frac{1}{2}$
256	$\frac{1}{2}$
338	1/ 3
344	$\frac{1}{2}$

ijk	d_{ijk}
355	$\frac{1}{2}$
366	$-\frac{1}{2}$
377	$-\frac{1}{2}$
448	$-(1/2\sqrt{3})$
558	$-(1/2\sqrt{3})$
668	$-(1/2\sqrt{3})$
778	$-(1/2\sqrt{3})$
888	$-(1/\sqrt{3})$.

(8.3.11)

It is usual to normalize the coupling constants F and D

$$D + F = 1 , \quad (8.3.12)$$

$$g_{\pi NN} = g(D + F) . \quad (8.3.13)$$

We may now deduce that, due to its conservation, the vector current is of the pure F type in the limit of exact SU(3) (i.e. where all SU(3)-violating interactions do not exist). The axial vector current, however, must be divided into an F- and a D-coupling.

We now examine the matrix elements for V_{ir} and A_{ir} between different states within SU(3) representations. Obviously we must concentrate on the octets, since these are the best-known of the supermultiplets . According to the Wigner-Eckart theorem (7), the matrix element between two octet states O_j and O_k is given by

$$\langle O_k | V_{ir} , A_{ir} | O_j \rangle = j f_{ijk} F_r^{V,A} + d_{ijk} D_r^{V,A} , \quad (8.3.14)$$

where $F_r^{V,A}$ and $D_r^{V,A}$ are reduced matrix elements. We assume

$$q = 0 \quad (8.3.15)$$

and hence our matrix elements F_r^V , F_r^A , D_r^V and D_r^A each contain only one form factor. Making an explicit matrix element calculation, we find that we have six arbitrary parameters in the final expression: a, b, a', b', $g_A^F(0)$ and $g_A^D(0)$. a and b are basically vector coupling constants, and similarly a' and b' are axial vector ones. In order to reduce the number of arbitrary constants, we now assume a hypothesis known as 'parallelism'. We denote the field operators for the three quarks by "A", "B" and "C".

If we impose the condition that quarks and leptons must enter symmetrically into the total weak current, then the latter becomes

$$j\, a\, \text{"}\bar{A}\text{"}\, \gamma_r\, (1 + \gamma_5)\, \text{"B"} \;+\; j\, b\, \text{"}\bar{A}\text{"}\, \gamma_r\, (1 + \gamma_5)\, \text{"C"} \;+$$

$$+\; j\, \bar{v}_e\, \gamma_r (1 + \gamma_5)\, e \;+\; j\, \bar{v}_\mu\, \gamma_r (1 + \gamma_5)\, \mu \;, \qquad (8.3.16)$$

where lepton symbols represent wave functions. From (8.3.16), we see that, if the quark model is indeed correct, then

$$b'/a' \;=\; b/a \;. \qquad (8.3.17)$$

We now recall the Cabibbo hypothesis from 5.6. First, we attempt to justify the condition (5.6.5). If (5.6.5) is true, then

$$J_r \;=\; \cos\theta\, (J_{1r} \;+\; j\, J_{2r}) \;+\; \sin\theta\, (J_{4r} \;+\; j\, J_{5r}) \;, \qquad (8.3.18)$$

$$J_{ir} \;=\; V_{ir} \;+\; A_{ir} \;. \qquad (8.3.19)$$

Since (8.3.18) belongs to an SU(3) octet, it is possible to perform a transformation in SU(3) space under which its $\Delta Y = 1$ component will vanish (8). This transformation is found to be

$$e^{2j\,\theta\, F_7} \;, \qquad (8.3.20)$$

which is equivalent to a rotation about the 7-axis in SU(3) space. We must rotate through the angle 2θ because the components of (8.3.18) form a U-spin doublet, and transform into each other by rotation through the angle θ. We note that the charge operator

$$Q \;=\; F_3 \;+\; (1/\sqrt{3})\, F_8 \qquad (8.3.21)$$

commutes with (8.3.20) and is hence invariant under this transformation. If

$$b/a \;=\; \tan\theta \;, \qquad (8.3.22)$$

then the octet commutation relation (see 8.4)

$$\left[F_i(t),\, J_{jr}(\underline{x},\, t)\right] \;=\; j\, f_{ijk}\, J_{kr}(\underline{x},\, t) \qquad (8.3.23)$$

yields

$$\exp\,(2j\,\theta\, F_7)\,\left(a(J_{1r} \;+\; j\, J_{2r}) \;+\; b(J_{4r} \;+\; j\, J_{5r})\right) \exp\,(-2j\,\theta\, F_7) \;=$$

$$=\; \sqrt{a^2 + b^2}\;\, (J_{1r} \;+\; j\, J_{2r}) \;. \qquad (8.3.24)$$

Since it is thought that the strong interaction does not discriminate between different directions in SU(3) space, the $\Delta Y = 0$ current on the right-hand side of (8.3.24) should have the same strength as the lepton currents in (8.3.16), which, by the CVC hypothesis, are of the same strength as $(J_{1r} + j J_{2r})$, so that

$$a^2 + b^2 = 1 . \qquad (8.3.25)$$

Thus we finally are left with three parameters: θ, $g_A^F(0)$ and $g_A^D(0)$, so that the matrix elements are

$$\langle B_k | V_r | B_j \rangle = (j/(2\pi)^3)(\sqrt{((m_j m_k)/(p_0 p_0'))}) \times$$
$$\bar{u}(p')\gamma_r (j \cos\theta \, f_{1+i2,\,jk} + j \sin\theta \, f_{4+i5,jk}) u(p) , \qquad (8.3.26)$$

$$\langle B_k | A_r | B_j \rangle = (j/(2\pi)^3)(\sqrt{((m_j m_k)/(p_0 p_0'))}) \times$$
$$\bar{u}(p')\gamma_r\gamma_5((j \cos\theta \, f_{1+i2,jk} + j \sin\theta \, f_{4+i5,jk}) g_A^F(0)$$
$$(\cos\theta \, d_{1+i2,jk} + \sin\theta \, d_{4+i5,jk}) g_A^D(0)) u(p) . \qquad (8.3.27)$$

We now discuss various tests of the 'octet current' hypothesis. We append a table of the matrix elements for beta decays predicted by (8.3.26) and (8.3.27):

Hadrons in decay	$\langle B_k \| V_r \| B_j \rangle$	$\langle B_k \| A_r \| B_j \rangle$
$n \longrightarrow p$	$\cos\theta$	$\cos\theta(F+D)$
$\Sigma \longrightarrow \Lambda$	0	$(2/\sqrt{6}) \cos\theta \, D$
$\Xi^- \longrightarrow \Xi^\circ$	$-\cos\theta$	$\cos\theta(-F+D)$
$\Lambda \longrightarrow p$	$-(3/\sqrt{6}) \sin\theta$	$(1/\sqrt{6}) \sin\theta(-3F-D)$
$\Sigma^- \longrightarrow n$	$-\sin\theta$	$\sin\theta(-F+D)$
$\Sigma^\circ \longrightarrow p$	$-(1/\sqrt{2}) \sin\theta$	$(1/\sqrt{2}) \sin\theta(-F+D)$
$\Xi^- \longrightarrow \Lambda$	$(3/\sqrt{6}) \sin\theta$	$(1/\sqrt{6}) \sin\theta(3F-D)$
$\Xi^- \longrightarrow \Sigma^\circ$	$(1/\sqrt{2}) \sin\theta$	$(1/\sqrt{2}) \sin\theta \, (F+D)$
$\Xi^\circ \longrightarrow \Sigma^+$	$\sin\theta$	$\sin\theta \, (F+D)$

(8.3.28)

where F denotes $g_A^F(0)$ and D $g_A^D(0)$. Since

$$g_A(0)/g_V(0) \sim 1.22, \qquad (8.3.29)$$

$$D + F \sim 1.22. \qquad (8.3.30)$$

Thus, assuming

$$G_V^n/G_\mu \quad = \quad 0.978 \pm 0.002 \ , \tag{8.3.31}$$

we obtain

$$\sin\theta \quad = \quad 0.209 \pm 0.016 \ . \tag{8.3.31}$$

However, the value of (8.3.31) is very sensitive both to radiative corrections and to the so-called 'weak interaction cut-off' energy (see chapter 9), so that a better estimate for $\sin\Theta$ is that from observed hyperon decay rates: (9)

$$\sin\Theta \quad = \quad 0.24 \pm 0.01 \ . \tag{8.3.32}$$

Further experiments on hyperon decay rates yield

$$F \quad = \quad 0.43 \pm 0.04 \ , \tag{8.3.33}$$

$$D \quad = \quad 0.79 \pm 0.04 \ . \tag{8.3.34}$$

Using the values (8.3.32), (8.3.33) and (8.3.34) we may predict the rate for any hyperon decay. Comparison with experiment demonstrates that the Cabibbo three-parameter model is very satisfactory. For example, theory yields the rate for the beta decay

$$\Lambda^\circ \longrightarrow p + e + \bar{\nu}_e \tag{8.3.35}$$

as

$$0.32 \quad \times \quad 10^{-2} \ , \tag{8.3.36}$$

normalized to the neutron decay rate, and experiments give (10)

$$(0.32 \pm 0.05) \quad \times \quad 10^{-2} \ , \tag{8.3.37}$$

in excellent agreement with theory.

8.4 The Algebra of Currents.

We recall that when we arrived at the CVC hypothesis in 5.2, we identified the current V_r^0 with the isospin current:

$$V_r^0(x) \quad = \quad (J_{1r}(x) + j\, J_{2r}(x)) \ , \tag{8.4.1}$$

and the isovector component of the electromagnetic interaction:

$$V_{3r}(x) \quad = \quad J_{3r}(x) \ . \tag{8.4.2}$$

In the absence of electromagnetism, all three components of $V_{ir}(x)$ are exactly conserved, so that the generators of isospin rotations are given purely by

$$I_i \quad = \quad -j \int d^3x \ \ V_{i4}(x) \qquad (i = 1, 2, 3) \ , \tag{8.4.3}$$

which implies that these generators satisfy the equal-time commutation relations:

$$\left[I_i(t), I_j(t)\right] = j\epsilon_{ijk} I_k(t) \qquad (i = 1, 2, 3) \;, \tag{8.4.4}$$

where ϵ_{ijk} is the Levi-Civita symbol, such that

$$\epsilon_{ijk} = +1 \tag{8.4.5}$$

for ijk an even permutation of 123, and

$$\epsilon_{ijk} = -1 \tag{8.4.6}$$

if ijk is an odd permutation of 123. At this point, we note that the generators I_r are the infinitesimal generators of the isospin group SU(2), with basis matrices (1.7.18). We discussed them in 5.1. In the presence of electromagnetism, the components I_1 and I_2 of the isospin current are no longer conserved, although I_3 is unaffected by electromagnetism, resulting in the CVC hypothesis. Thus SU(2) is not a symmetry of the total weak Hamiltonian, due to radiative effects. If we now take $A_r^0(x)$ to be the (1 + j2) component of an isovector axial vector current $A_{ir}^0(x)$ in analogy to (8.4.1), (8.4.3) becomes ($I_i^5 = \gamma_5 I_i$)

$$I_i^5(t) = -j \int d^3x \; A_{i4}^5(\underline{x}, t) \;. \tag{8.4.7}$$

Since the $A_{ir}(x)$ are not conserved, the $I_i^5(t)$ are now time-dependent, and as $I_i^5(t)$ is an isovector, we obtain the equal-time commutation relations:

$$\left[I_i(t), I_j^5(t)\right] = j\,\epsilon_{ijk} I_k^5(t) \qquad (i = 1, 2, 3) \;. \tag{8.4.8}$$

We now make the assumption that, although (8.4.8) is, at present, only provable in the absence of electromagnetism, it also applies in the presence of electromagnetism. We then require

$$\left[I_i^5(t), I_j^5(t)\right] = j\,\epsilon_{ijk} I_k(t) \;, \tag{8.4.9}$$

although this has little justification. (8.4.4), (8.4.8) and (8.4.9) constitute the basic relations of SU(2) ⊗ SU(2) current algebra. Strictly, since we are concerned with time rather than space integrals, our above discussion should be known as 'charge' algebra. We note that (8.4.9) is the fundamental relation involved in the Adler-Weissberger formula (11), which may be used to calculate the axial vector coupling constant in neutron decay to an accuracy of up to 95% by means of the form factors involved in πN scattering. G_A may also be obtained by the Goldberger-Treiman relation (12) deduced from dispersion theory:

$$2m\,(g/\sqrt{2})\,G_A \quad = \quad -f\, g_{NN\pi}\,, \qquad (8.4.10)$$

where m is the nucleon mass, $g_{NN\pi}$ is the π-N coupling constant, f is a further strong interaction constant and g is the total weak coupling constant. To conclude our discussion of $SU(2) \otimes SU(2)$ charge algebra, we mention the so-called 'chiral' $SU(2) \otimes SU(2)$ algebra. By taking the linear combinations of I_i and I_i^5:

$$I_i^{L,R} \quad = \quad \tfrac{1}{2}(I_i \;\pm\; I_i^5)\,, \qquad (8.4.11)$$

we obtain

$$[I_i^{L,R}\,,\, I_j^{L,R}] \quad = \quad j\,\epsilon_{ijk}\, I_k^{L,R} \qquad (i = 1,\,2,\,3),\ (8.4.12)$$

$$[I_i^L\,,\, I_j^R] \quad = \quad 0 \qquad (i = 1,\,2,\,3).\ (8.4.13)$$

However, since $A_r(x)$ is not conserved, chiral $SU(2) \otimes SU(2)$ is not an exact symmetry of the weak Hamiltonian. It has, nevertheless, been suggested that the commutation relations (8.4.12) and (8.4.13) still hold good despite PCAC.

We now examine $SU(3) \otimes SU(3)$ current algebra. In the limit of exact SU(3) symmetry, there exist eight conserved currents $V_{ir}(x)$ $(i = 1, 2, 3, \ldots\ldots, 8)$, and thus the SU(3) generators are given by

$$F_i(t) \quad = \quad -j \int V_{i4}(\underline{x},\, t)\, d^3x \qquad (i = 1, \ldots, 8), \qquad (8.4.14)$$

satisfying the commutation relations

$$[F_i(t),\, F_j(t)] \quad = \quad j\, f_{ijk}\, F_k(t) \qquad (i = 1, \ldots, 8)\,, \qquad (8.4.15)$$

where the f_{ijk} were given in 8.1. The generators $F_i(t)$ are usually known as the vector charges. In exact SU(3) all the $F_i(t)$ are time-independent, but in the presence of the strong interaction, only F_1, F_2, F_3 and F_8 are time-independent, and when the electromagnetic interaction is also included, only

$$F_3 \quad = \quad I_3\,, \qquad (8.4.16)$$

$$F_8 \quad = \quad (\sqrt{3}/2)Y \qquad (8.4.17)$$

remain independent of time. At this point, we make the assumption that even when SU(3) symmetry is no longer exact, the relation (8.4.14) still remains true. As before, we assume that the $\Delta Y = 0$ and $\Delta Y = 1$ components of the current $V_{ir}(x)$ may be taken as the (1 + j2) and (4 + j5) parts of a vector current octet. Similarly, we postulate that the hypercharge-conserving and hypercharge-violating components of the axial vector hadron current are also the (1 + j2) and (4 + j5) parts of an axial vector current octet. Defining

$$F_i^5(t) = -j \int d^3x \; A_{i4}(\underline{x}, t) \qquad (i = 1,\dots,8) \; , \qquad (8.4.18)$$

we obtain

$$\left[F_i(t),\, F_j^5(t)\right] = j\, f_{ijk}\, F_k(t) \qquad (i = 1,8) \; , \qquad (8.4.19)$$

which is analogous to the statement (8.4.8) for SU(2) ⊗ SU(2) current algebra. Without proof, we adapt (8.4.9) for SU(3):

$$\left[F_i^5(t),\, F_j^5(t)\right] = j\, f_{ijk}\, F_k(t) \; . \qquad (8.4.20)$$

We now examine the so-called 'triplet' model for $V_{ir}(x)$ and $A_{ir}(x)$, in which the conditions (8.4.15) and (8.4.19) are fulfilled. We set (the first suffix is the unitary index, the second the Lorentz index)

$$V_{ir}(x) = j\, \widetilde{\psi}(x)\, \gamma_r\, (g_i/2)\, \psi(x) \; , \qquad (8.4.21)$$

$$A_{ir}(x) = j\, \widetilde{\psi}(x)\, \gamma_r \gamma_5\, (g_i/2)\, \psi(x) \; , \qquad (8.4.22)$$

where $\psi(x)$ is a unitary triplet

$$\psi(x) = \begin{bmatrix} \psi^1(x) \\ \psi^2(x) \\ \psi^3(x) \end{bmatrix} \qquad (8.4.23)$$

We find (13) that, if we may integrate over all of three-space, then the so-called 'Schwinger terms' dependent upon the gradient of the three-space normalization δ function vanish from the equal-time commutators of $\psi(x)$, so that only the relations

$$\left[F_i(t),\, V_{jm}(\underline{x}, t)\right] = j\, f_{ijk}\, V_{km}(\underline{x}, t) \; , \qquad (8.4.24)$$

$$\left[F_i(t),\, A_{jm}(\underline{x}, t)\right] = j\, f_{ijk}\, A_{km}(\underline{x}, t) \; , \qquad (8.4.25)$$

$$\left[F_i^5(t),\, V_{jm}(\underline{x}, t)\right] = j\, f_{ijk}\, A_{km}(\underline{x}, t) \; , \qquad (8.4.26)$$

$$\left[F_i^5(t),\, A_{jm}(\underline{x}, t)\right] = j\, f_{ijk}\, V_{km}(\underline{x}, t) \qquad (8.4.27)$$

remain. The equations (8.4.24), (8.4.25), (8.4.26) and (8.4.27) hold for any triplet model, such as the quark, Sakata (14) or Maki-Hara (15) models. The Sakata model postulates that the observed hadrons are bound states of the 'fundamental' particles p, n and Λ°. The Maki-Hara model is based on the same principle, but assumes the 'fundamental' particles to be Ξ°, Ξ^-, and Λ°. However, the commutation relations of the $\underline{x}$ (space) components of the axial vector and vector currents with the electromagnetic current do tend to vary from model to model. The reason for this is that the electromagnetic current itself:

$$J_r^{em} = a_1 V_{3r} + a_2 (1/\ 3) V_{8r} + a_3 \sqrt{(2/3)}\, V_{0r} \tag{8.4.28}$$

contains three arbitrary constants a_1 , a_2 and a_3 . We now append a table giving the values of these parameters for different models:

Model	Charge on triplet			a_1	a_2	a_3
Quark (Q_1 , Q_2 , Q_3)	2/3	-1/3	-1/3	1	1	0
Sakata (p, n, Λ)	1	0	0	1	1	1
Maki-Hara (Ξ°, Ξ^-, Λ°)	0	-1	0	1	-1	-1
Marshak (16) (Ξ°, Ξ^-, Ω^-)	0	-1	-1	1	1	-2

At present, there is no way to ascertain the constants a_i , and thus it is not possible to discriminate between the various triplet models proposed. At this point, we mention the scalar and pseudoscalar current densities equivalent to (8.4.21) and (8.4.22):

$$S_i(x) = \widetilde{\psi}(x)\, g_i/2\ \widetilde{\psi}(x)\ , \tag{8.4.29}$$

$$P_i(x) = j\widetilde{\psi}(x)\gamma_5\, (g_i/2)\ \psi(x)\ . \tag{8.4.30}$$

The associated equal-time commutation relations are

$$\left[F_i^5(t),\ S_j(\underline{x},\ t)\right] = j\, d_{ijk}\, P_k(\underline{x},\ t)\ , \tag{8.4.31}$$

$$\left[F_i^5(t),\ P_j(\underline{x},\ t)\right] = -\,j\, d_{ijk}\, S_k(\underline{x},\ t)\ , \tag{8.4.32}$$

$$\left[F_i(t), S_j(\underline{x}, t)\right] = j f_{ijk} S_k(\underline{x}, t), \qquad (8.4.33)$$

$$\left[F_i(t), P_j(\underline{x}, t)\right] = j f_{ijk} P_k(\underline{x}, t), \qquad (8.4.34)$$

using a triplet model. It has been suggested that the scalar and pseudoscalar interactions may contribute a few terms to the hadronic Hamiltonian, but there is good evidence to show that their amplitude must be very small (see Chapter 6).

8.5 Applications of SU(3) Symmetry in Hyperon Decays.

Since the amplitude for

$$\Delta I = 3/2 \qquad (8.5.1)$$

decays is definitely nonzero, we are forced to conclude that the hadronic $\Delta Y = 1$ Hamiltonian H^h receives contributions not only from octet currents, but also from currents transforming as members of the representation $\{27\}$ formed in

$$\{8\} \otimes \{8\} = \{1\} \oplus \{8_S\} \oplus \{8_A\} \oplus \{10\} \oplus \{10^*\} \oplus \{27\}, \qquad (8.5.2)$$

where the index S denotes a representation formed by the symmetric combination of the components of the representations $\{8\}$ on the left-hand side of the reduction, and A one formed by their antisymmetric combination. The usual current octet is of the form $\{8_S\}$. The octet current hypothesis implies that only octet currents exist, and hence, if this is correct, then there must exist some mechanism for enhancing octet currents over 27-plet ones. We now examine some of the phenomenological predictions of the octet current theory. We write a typical hadron decay as

$$B \longrightarrow B' + \pi + S, \qquad (8.5.3)$$

where S is a spinless spurion with zero four-momentum. With the formalism (8.5.3) we may write all interactions as SU(3) invariants, and, by considering the properties of the spurion, we may deduce those of the Hamiltonian responsible for the reaction. We represent our octet spurion by the Hermitean 3 X 3 matrix S_i $(i = 1, 8)$. Since the spurion has the same transformational properties as the Hamiltonian H^h, we must examine the behaviour of components of current octets under the charge conjugation operator in order to ascertain its effect on the spurion. Most octets O_i are self-conjugate under the C operator, and hence

$$C(O_i) \longrightarrow P_C \, \epsilon_i \, O_i \quad , \qquad (8.5.4)$$

where

$$\epsilon_i \;=\; +1 \qquad (i = 1, 3, 4, 6, 8) \; , \qquad (8.5.5)$$

$$\epsilon_i \;=\; -1 \qquad (i = 2, 5, 7) \; , \qquad (8.5.6)$$

P_C denoting the C-parity of the neutral components,3 and 8,of O_i (17). (8.5.4) may be deduced from the behaviour of the generators F_i under the matrix transposition operator. Thus the spurion may have either $P_C = +1$ or -1. Similarly, since both parity-conserving (p.c.) and parity-violating (p.v.) weak hadron decays do occur, the spurion may also have either even or odd parity, P. Initially, therefore, we must consider four cases for the spurion parities:

$$P = -1 \;\; (\text{p.v.}) \qquad C = +1 \; , \qquad (8.5.7)$$

$$P = -1 \;\; (\text{p.v.}) \qquad C = -1 \; , \qquad (8.5.8)$$

$$P = +1 \;\; (\text{p.c.}) \qquad C = +1 \; , \qquad (8.5.9)$$

$$P = +1 \;\; (\text{p.c.}) \qquad C = -1 \; . \qquad (8.5.10)$$

Since CP invariance holds to a high degree in the weak interaction (see Chapter 7), we find that H^h and hence S must have

$$CP = +1 \; . \qquad (8.5.11)$$

(8.5.11) requires that the spurion in the case (8.5.7) transforms as g_7 under SU(3), since

$$S_7 \xrightarrow{\;CP\;} -\epsilon_7 \, S_7 \; . \qquad (8.5.12)$$

Similarly, the spurion in (8.5.8) transforms like g_6 , that in (8.5.9) also as g_6 , and that in (8.5.10) as g_7 . We thus consider the cases (8.5.8) and (8.5.9), in which the Hamiltonian is of the type g_6 . Thus the matrix element for these processes is

$$\langle B^a_b \, P^c_d \mid g_6 \mid B^e_f \rangle \; , \qquad (8.5.13)$$

using the matrix notation for the baryons and mesons involved. The most general form of the matrix element (8.5.13) may be obtained by evaluating traces (see 2.8) in the product

$$\bar{B}^a_b \;\; B^c_d \;\; P^e_f \;\; S^g_h \qquad (8.5.14)$$

while the indices in (8.5.14) are permuted in all possible ways. Since all the SU(3) octets must have vanishing trace, the general matrix element (8.5.13) may be written

$$M = \sum_{i=1}^{9} C_i' \; M_i \, , \tag{8.5.15}$$

where

$$M_1 = \mathrm{Tr}\,(\bar{B}\; B\; P\; S) \tag{8.5.16}$$

$$M_2 = \mathrm{Tr}\,(\bar{B}\; S\; B\; P) \tag{8.5.17}$$

$$M_3 = \mathrm{Tr}\,(\bar{B}\; P\; S\; B) \tag{8.5.18}$$

$$M_4 = \mathrm{Tr}\,(\bar{B}\; P\; B\; S) \tag{8.5.19}$$

$$M_5 = \mathrm{Tr}\,(\bar{B}\; B\; S\; P) \tag{8.5.20}$$

$$M_6 = \mathrm{Tr}\,(\bar{B}\; S\; P\; B) \tag{8.5.21}$$

$$M_7 = \mathrm{Tr}\,(\bar{B}\; P)\;\; \mathrm{Tr}\,(B\; S) \tag{8.5.22}$$

$$M_8 = \mathrm{Tr}\,(\bar{B}\; S)\;\; \mathrm{Tr}\,(B\; P) \tag{8.5.23}$$

$$M_9 = \mathrm{Tr}\,(\bar{B}\; B)\;\; \mathrm{Tr}\,(P\; S) \, , \tag{8.5.24}$$

and the C_i' are scalar coefficients. However, we may decompose our matrix element still further. Each term (8.5.16) through (8.5.23) consists of a p and an s wave component, so that, for example,

$$M_6 = M_{6s} + M_{6p} = \mathrm{Tr}\,(\bar{B}\; B\; P\; S) + \mathrm{Tr}\,(\bar{B}\; \gamma_5\; B\; P\; S) \; . \tag{8.5.25}$$

It has been demonstrated that (18)

$$M_7 + M_8 + M_9 = \sum_{i=1}^{6} M_i \, , \tag{8.5.26}$$

and, by an explicit calculation, we may verify that none of the traces M_5 , M_6 , M_7 , or M_9 contribute to observed processes, so that there exist five linearly independent coupling constants C_i' in the general hadronic matrix element. We now recall the effect of the C operator on the various components of our matrix element:

$$B^a_b \xrightarrow{\;\;C\;\;} \bar{B}^b_a \, , \tag{8.5.27}$$

$$P^c_d \xrightarrow{\;\;C\;\;} \bar{P}^d_c \, , \tag{8.5.28}$$

$$S_6 \xrightarrow{\;\;C\;\;} -\epsilon_6\, S_6 = -S_6 \, , \tag{8.5.29}$$

and under C we may show that

$$M_1 \longleftrightarrow M_5 \tag{8.5.30}$$

$$M_3 \longleftrightarrow M_6 \tag{8.5.31}$$

$$M_7 \longleftrightarrow M_8 \tag{8.5.32}$$

$$M_2 \longleftrightarrow M_2 \tag{8.5.33}$$

$$M_4 \longleftrightarrow M_4 \tag{8.5.34}$$

$$M_9 \longleftrightarrow M_9 \; . \tag{8.5.35}$$

Writing the decay Hamiltonian in the standard form

$$(\bar{B}^a_b(A \;+\; B\gamma_5)\, B^c_d)\, P^e_f \;, \tag{8.5.36}$$

we see that the term in the coupling constant A has odd parity, whereas that in B has even parity. Imposing rigid CP invariance, we may deduce that only the combinations

$$M_{1s} \;-\; M_{5s} \tag{8.5.37}$$

$$M_{1p} \;+\; M_{5p} \tag{8.5.38}$$

$$M_{3s} \;-\; M_{6s} \tag{8.5.39}$$

$$M_{3p} \;+\; M_{6p} \tag{8.5.40}$$

$$M_{7s} \;-\; M_{8s} \tag{8.5.41}$$

$$M_{7p} \;+\; M_{8s} \tag{8.5.42}$$

$$M_{2p} \tag{8.5.43}$$

$$M_{4p} \tag{8.5.44}$$

ever enter into the matrix element, and always separately. (8.5.37) through (8.5.43) in turn indicate that

$$a_2 \;=\; a_4 \;=\; a_9 \;=\; 0 \tag{8.5.45}$$

$$a_1 \;=\; -a_5 \tag{8.5.46}$$

$$a_3 \;=\; -a_6 \tag{8.5.47}$$

$$a_7 \;=\; -a_8 \tag{8.5.48}$$

$$b_1 \;=\; b_5 \tag{8.5.49}$$

$$b_3 \quad = \quad b_6 \tag{8.5.50}$$

$$b_7 \quad = \quad b_8 \ , \tag{8.5.51}$$

where a_i and b_i are the coupling constants for the parity-conserving and parity-violating components of the total matrix element respectively. Thus the total parity-violating matrix element is of the form

$$a_1 \left(\mathrm{Tr}(\bar{B}\, B\, P\, g_6) - \mathrm{Tr}(\bar{B}\, B\, g_6\, P)\right) + a_3(\mathrm{Tr}(\bar{B}\, P\, g_6\, P) - \mathrm{Tr}(\bar{B}\, g_6\, P\, B)) +$$

$$+ \ a_7(\mathrm{Tr}(\bar{B}\, P)\, \mathrm{Tr}(B\, g_6) - \mathrm{Tr}(\bar{B}\, g_6)\, \mathrm{Tr}(B\, P)\ ) \ , \tag{8.5.52}$$

and the parity-conserving one of the type

$$b_1 \left(\mathrm{Tr}(\bar{B}\ {}_5\, B\, P\, g_6) + \mathrm{Tr}(\bar{B}\, \gamma_5\, B\, g_6\, P)\right) + b_2\, \mathrm{Tr}(\bar{B}\, \gamma_5\, g_6\, B\, P) +$$

$$+ \ b_3(\mathrm{Tr}(\bar{B}\, \gamma_5\, P\, g_6\, B) + \mathrm{Tr}(\bar{B}\, \gamma_5\, g_6\, P\, B)) + b_4(\bar{B}\, \gamma_5\, P\, B\, g_6) +$$

$$+ \ b_7(\mathrm{Tr}(\bar{B}\, P)\, \gamma_5\, \mathrm{Tr}(B\, g_6) + \mathrm{Tr}(B\, g_6)\, \gamma_5\, \mathrm{Tr}(\bar{B}\, P)\ ) \ . \tag{8.5.53}$$

Thus far, we have tacitly assumed that decays of the types (8.5.7) and (8.5.10) do not occur, but we could equally have formulated matrix elements on the hypothesis that the spurion transforms as g_7 under SU(3). However, there exists good evidence in favour of a low CP-violating amplitude in the weak interaction, and hence, although rare, it is still possible for the spurion to be of the form g_7 rather than g_6 .

Using (8.5.52) and (8.5.53), we obtain the following relations for the parity-violating and parity-conserving amplitudes

$$A(\Lambda^0_-) \quad = \quad -\sqrt{2}\, A(\Lambda^0_0) \quad = \quad 1/\sqrt{6}\, a_1 - \sqrt{(2/3)}\, a_3 \tag{8.5.54}$$

$$A(\Sigma^+_+) \quad = \quad -\, a_7 \tag{8.5.55}$$

$$A(\Sigma^-_-) \quad = \quad a_1 - a_7 \tag{8.5.56}$$

$$A(\Sigma^+_0) \quad = \quad -\, 1/\sqrt{2}\ a_1 \tag{8.5.57}$$

$$A(\Xi^-_-) \quad = \quad -\ 2\, A(\Xi^0_0) \quad = \quad -\sqrt{(2/3)}\, a_1 + \sqrt{(1/6)}\, a_3 \tag{8.5.58}$$

$$B(\Lambda^0_-) \quad = \quad -\sqrt{2}\, B(\Lambda^0_0) \quad = \quad -\, 1/\sqrt{6}\, (b_3 - b_4) - 1/\sqrt{6}\, (b_3 - b_1) \tag{8.5.59}$$

$$B(\Sigma^+_+) \quad = \quad b_4 + b_7 \tag{8.5.60}$$

$$B(\Sigma^-_-) \quad = \quad b_1 + b_7 \tag{8.5.61}$$

$$B(\Sigma^-_-) = b_1 + b_7 \tag{8.5.62}$$

$$B(\Sigma^+_0) = 1/\sqrt{2}\,(b_4 - b_1) \tag{8.5.63}$$

$$B(\Xi^-_-) = -\sqrt{2}\,B(\Xi^0_0) = 1/\sqrt{6}\,(b_3 - b_1) - 1/\sqrt{6}\,(b_1 - b_2)\,, \tag{8.5.64}$$

where the suffixes on the left-hand side denote the chargeson the decay pions. Solving for the amplitudes a_i in (8.5.54) through (8.5.58), we obtain the Lee-Sugawara (L-S) (19) relation for the s-wave amplitudes:

$$2A(\Xi^-_-) + A(\Lambda^0_-) = \sqrt{3}\,A(\Sigma^+_0)\ . \tag{8.5.65}$$

Similarly, solving in (8.5.59) through (8.5.64), we find that

$$2B(\Xi^-_-) + B(\Lambda^0_-) = 3\,B(\Sigma^+_0)\ . \tag{8.5.66}$$

Experimentally,

$$(2\Xi^-_- + \Lambda^0_-)/\sqrt{3}: \quad A = -1.440 \pm 0.037, \quad B = 14.03 \pm 0.657\ , \tag{8.5.67}$$

$$\Sigma^+_0: \quad A = -1.155 \pm 0.187 \quad B = 15.713 \pm 1.42\ , \tag{8.5.68}$$

in good agreement with the Lee-Sugawara relations (8.5.65) and (8.5.66). There exist a number of alternative methods for deducing (8.5.65). The first employs dispersion theory and a number of properties of the octet. In addition to the L-S relation, this approach yields

$$A(\Sigma^-_-)/A(\Lambda^0_-) = (\sqrt{2}/\sqrt{3})\,(m_\Sigma - m_N)/(m_\Lambda - m_N) = 1.19 \tag{8.5.69}$$

$$A(\Xi^-_-)/A(\Lambda^0_-) = -(m_\Xi - m_N)/(m_\Lambda - m_N) = -1.23 \tag{8.5.70}$$

$$A(\Sigma^+_+) = 0\ . \tag{8.5.71}$$

Experiments give (20)

$$A(\Sigma^-_-)/A(\Lambda^0_-) = 1.203 \pm 0.708\ , \tag{8.5.72}$$

$$A(\Xi^-_-)/A(\Lambda^0_-) = -1.307 \pm 1.208\ , \tag{8.5.73}$$

$$A(\Sigma^+_+) = 0.016 \pm 0.034\ , \tag{8.5.74}$$

in good agreement with the predictions of dispersion theory. Current algebra constitutes an alternative approach to the theory of weak hyperon decay. Using the Born approximation (see 5.4), we perform a number of explicit calculations according to the octet current hypothesis. We then calculate the contribution to the Hamiltonian from currents transforming as components of a 27-plet. This yields

$$A(\Lambda^0_-) + 2\,A(\Xi^-_-) = \sqrt{3}\,A(\Sigma^+_0) + \sqrt{(3/2)}\,A(\Sigma^+_+), \qquad (8.5.75)$$

which is equivalent to the Lee-Sugarawa relation (8.5.65) if and only if (8.5.71) is correct.

CHAPTER NINE: THE INTERMEDIATE VECTOR BOSON HYPOTHESIS.

9.1 The Non-Local Theory of the Weak Interaction.

In 1935, Yukawa (1) postulated that, in analogy to his theory of the strong interaction, weak beta decay might occur via an intermediate particle, which he denoted by W. Thus nuclear beta decay would be a two-stage process:

$$(A, Z) \longrightarrow W^- + (A, Z+1) \longrightarrow e^- + \bar{\nu}_e + (A, Z+1) \, , \tag{9.1.1}$$

so that the W particle itself must undergo beta decay. Following its discovery in 1936 (see 4.1), it was immediately suggested that the muon could be identified with the W particle, since it beta decayed in the required manner (2). However, in 1947 it was shown that muons were produced predominantly in the decay (see 6.1)

$$\pi \longrightarrow \mu + \bar{\nu}_\mu \, , \tag{9.1.2}$$

implying that the pion was initially the intermediate particle in beta decay. Moreover, the pion was already thought to be the mediator of the strong interaction, and thus its properties were incompatible with those predicted for the W particle. In Chapter Six, we saw that the pion does contribute to the coupling constant in hypercharge-conserving weak processes via the form factors, but these vanish when

$$q = 0 \, , \tag{9.1.3}$$

and the weak coupling constant does not. Nevertheless, it is possible to account for particular weak hadronic processes in terms of the vector mesons ρ , A_1 , K^* and so on, but this hypothesis predicts incorrect selection rules.

In (4.4.33) we demonstrated that the cross-section for electron-neutrino scattering was given by

$$\sigma_{\nu e} = \sigma_0 \, (2E^2)/(1 + 2E) \, , \tag{9.1.4}$$

so that

(9.1.5)

as

$$E \longrightarrow \infty \,, \tag{9.1.6}$$

in clear contradiction with experimental facts (3). In obtaining (9.1.4), we made two major assumptions: first, the weak interaction is local, i.e. it occurs at a single distinct point in space-time; and second, it is correct to apply first-order perturbation theory. From the unitarity of the S-matrix[1], we may deduce that the upper limit on the e-v cross-section is of order

$$4\pi \; \lambda^2/2 \quad , \tag{9.1.7}$$

where λ is the de Broglie wavelength of the incident particle. Thus, at a given critical or 'cut-off' energy, the formula (9.1.4) will violate the unitarity of the matrix element. At this energy, making an approximation in the extreme relativistic limit,

$$\sigma \quad = \quad (G^2 \, E^2_{cr})/\pi \quad = \quad (4\pi \lambda^2_{cr})/2 \, , \tag{9.1.8}$$

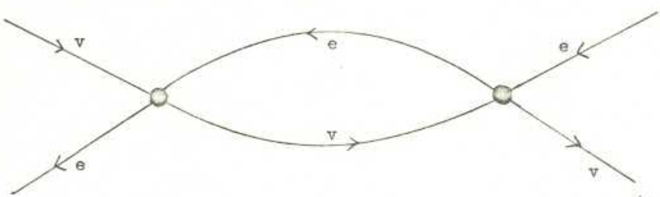
so that

$$E_{cr} \quad = \quad \sqrt{((2\sqrt{2}\pi)/G)} \quad = \quad \sqrt{(2\sqrt{2}\pi \;\; 10^5)} \; m \quad \sim \quad \sim \quad 10^3 \text{ GeV} \, . \tag{9.1.9}$$

In 1936 Heisenberg pointed out that (4) that at energies

$$E \;\sim\; E_{cr} \quad , \tag{9.1.10}$$

first-order perturbation theory is no longer applicable, and hence we must take into account such diagrams as



(9.1.11)

However, the loop in (9.1.11) and other associated diagrams gives rise to a divergenceless integral, yielding an infinite value. This fact is, at present, an unsolved problem in field theory, although Weinberg's suggestion (5) that they may be avoided by postulating a neutral W particle appears plausible. Nevertheless, we see that diagrams such as (9.1.11) imply a definite non-local weak interaction, since there are two weak vertices which are spatially separated. E_{cr} corresponds, according to the uncertainty relation

$$\Delta p \cdot \Delta s \geqslant \hbar , \qquad (9.1.12)$$

to a length of about 10^{-19} m, which is thus assumed to be the lower limit of the effective range of the weak interaction. A further possibility is that the fundamental four-fermion coupling is itself non-local, and in fact has a range of up to 10^{-15} m. A larger range than this may be excluded because of muon decay data. One suggestion is that normal spacetime laws cease to be applicable at distances smaller than about 10^{-15} m, although this hypothesis appears unlikely in view of a number of experiments performed to check the validity of quantum electrodynamics (electromagnetic field theory) over small distances. Thus we are forced to conclude that Yukawa's hypothesis for the weak interaction is essentially correct, and that there must exist an intermediate boson which is responsible for weak interactions.

We write our new interaction Hamiltonian as (6)

$$H_I = g_W J_r(x) \bar{W}_r(x) + \text{Herm. conj.}, \qquad (9.1.13)$$

where $J_r(x)$ is the standard weak current and $W_r(x)$ is the charged W particle wave function. Since the prescence of an intermediate particle implies that all observed weak processes are second-order 'semiweak' processes, the basic coupling of the W particle with the weak current must be of order $\sqrt{G}_W$, where G_W is the normal first-order weak coupling constant. Recalling the definition (2.6.22), we find that the second-order effective Hamiltonian arising from (9.1.13) is given by

$$H_I = -j g_W^2 \int d^3x' \; \Delta^W_{q,r} (x - x') \, T \, (J_r(x) \, \bar{J}_q(x')), \qquad (9.1.14)$$

which is simply a non-local version of the standard current-current Hamiltonian

$$H_I = - G/2 \; \bar{J}_r(x) \, J_r(x) \, . \qquad (9.1.15)$$

In (9.1.14), the term

$$\Delta^W_{q,r} (x - x') \, T \qquad (9.1.16)$$

is known as a propagator. Its basic effect is to create a W particle at a point x, and to destroy it again at x'. The matrix element for the process is T. We find that, with a suitable choice of quantum numbers for the W particle, the current-current and intermediate boson Hamiltonians are identical in the limit (9.1.3) if and only if their coupling constants obey the relation

$$(g_W^2/m_W^2) = G/\sqrt{2} . \qquad (9.1.17)$$

Thus the existence of the W particle will become important in weak processes only when the momentum transferred exceeds m_W, the mass of the W particle.

Since the weak current

$$J_r^W(x) = L_r(x) + J_r(x) = L_r(x) + \cos\theta\, J_r^0(x) + \sin\theta\, J_r^1(x) \qquad (9.1.18)$$

to which we assume the W particle particle to couple is charged, we are forced to conclude that the W must also be charged. Furthermore, the weak current is a Lorentz vector, and hence the W must also be a vector, resulting in its alternative name: the intermediate vector boson (IVB). However, there have also been attempts to show that the W particle is (7) a scalar rather than a vector particle. We write the muon decay Hamiltonian

$$H_I^N = -G/2\,(\bar{e}\,\gamma_r\,(1 + \gamma_5)\mu)\,(\bar{v}_\mu\,\gamma_r\,(1 + \gamma_5)\,v_e) + $$
$$+ \text{ Herm. conj.,} \qquad (9.1.19)$$

or, applying the Fierz reordering matrix (3.3.40),

$$H_I^N = -G/2\,(\mu^c\,\gamma_r(1 + \gamma_5)\,v_e)\,(\bar{v}_\mu\,(1 - \gamma_5)\,e^c) + $$
$$+ \text{ Herm. conj.} \qquad (9.1.20)$$

However, the interaction (9.1.20) may also arise in second order from an intermediate scalar boson interaction:

$$H_I^{ISB} = g_1\,\mu^c\,(1 + \gamma_5)\,v_e\,B + g_2\,\bar{v}_\mu\,(1 - \gamma_5)\,e^c\,B^+ + $$
$$+ \text{ Herm. conj.,} \qquad (9.1.21)$$

where B denotes the scalar boson wave function. (9.1.21) yields the same matrix element as (9.1.20) if and only if

$$(g_1\,g_2)/m_B^2 = G/\sqrt{2} . \qquad (9.1.22)$$

One advantage of the ISB theory over the IVB one is that the former is renormalizable[2], whereas the latter is not. However, the ISB hypothesis requires a nonzero lepton number assignment for the W particle, and, unless we accept the existence of more than one type of intermediate boson, does not allow a satisfactory unification of all weak processes.

9.2 Effects of the W Particle on Weak Processes.

We first consider the pure leptonic weak interactions. We note that the Hamiltonian (9.1.14) predicts self-current terms in the leptonic weak interaction of the same strength as the muon decay term. Failure to observe self-current processes would constitute a good argument against the existence of the IVB. The usual semiweak leptonic Hamiltonian is written

$$H_I = g_W (L_r^e + L_r^\mu) \bar{W}_r \quad \text{Herm. conj.} \tag{9.2.1}$$

However, there is now no reason to assume that no derivatives of the basic lepton fields occur in the Hamiltonian, and hence we may write, for example,

$$\begin{aligned} H_I^d = {} & g_W^S (\bar{v}_e(x)(a_1 + b_1 \gamma_5) e(x)) \partial/\partial x_r \bar{W}_r(x) + \\ & + g_W^V (\bar{v}_e(x) \gamma_r (1 + \gamma_5) e(x)) \bar{W}_r(x) + \\ & + \tfrac{1}{2} g_W^T (\bar{v}_e(x) \sigma_{rq} (a_2 + b_2 \gamma_5) \bar{e}(x)) (\partial/\partial x_r \bar{W}_q(x) - \partial/\partial x_q \bar{W}_r(x)) \end{aligned} \tag{9.2.2}$$

including scalar and tensor as well as vector lepton currents. The effect of the derivative couplings in (9.2.2) is to introduce momentum-dependent terms of the form $\sigma_{qr} q_q$ and q_r ($\sigma_{\alpha\beta}$ is the gamma matrix anticommutator) in addition to the pure vector γ_r term into the electron and muon current matrix elements. However, since most weak processes involve small coupling constants and low momentum-transfer, q, the derivative terms in the Hamiltonian will be almost unobservable, and hence we usually disregard them.

We now discuss the effect of the IVB on the asymmetry parameters in muon decay. Recalling ρ (4.2.26) and ξ (4.2.28), an explicit calculation yields the corrected values of these parameters as

$$\rho = \rho_0 (1 + (4/9) (m_\mu / m_W)^2) , \tag{9.2.3}$$

$$\xi = \xi_0 (1 - (2/5) (m_\mu / m_W)^2) , \tag{9.2.4}$$

where ρ_0 and ξ_0 are their values assuming point interaction. Taking the tentative experimental estimate

$$m_W \sim 2 \text{ GeV} , \qquad (9.2.5)$$

we find that

$$\rho = \rho_0 (1 + 2.8 \times 10^{-6}) , \qquad (9.2.6)$$

$$\xi = \xi_0 (1 - 2.5 \times 10^{-6}) , \qquad (9.2.7)$$

so that the effects of the IVB in muon decay are unobservable. A further, more sensitive, test of the IVB hypothesis is afforded by studying the rates of processes such as

$$\nu_e + e^- \longrightarrow \nu_e + e^- , \qquad (9.2.8)$$

$$\nu_\mu + e^- \longrightarrow \nu_e + \mu^- . \qquad (9.2.9)$$

As we saw above (9.1.4), the local weak interaction theory predicts infinite cross-section for these reactions at high energy. The IVB model, however, yields

$$\sigma^{IVB} = (8\, g_W^4\, p^2)/(\pi\, m_W^2 (m_W^2 + 4p^2)) , \qquad (9.2.10)$$

where p is the incident particle momentum. (9.2.10) predicts

$$\sigma^{IVB} \longrightarrow G^2/\pi \quad m_W^2 \qquad (9.2.11)$$

as

$$p \longrightarrow \infty , \qquad (9.2.12)$$

in agreement with the preliminary results obtained in high-energy neutrino-scattering experiments at NAL Batavia (8).

In the neutron decay matrix element (5.5.3), (5.5.4), the extra term added because of the IVB is undetectable, since we do not know enough about the form factors involved. Furthermore, at high q^2, the form factors decrease considerably, reducing the value of the matrix element significantly, thus rendering accurate measurement very difficult. The pure hadron processes are also comparatively insensitive to the possible existence of the IVB. However, using the soft pion and soft kaon approximations[3], the Weinberg sum rules[4] and PCAC, we may derive an expression for the T matrix element in the process

$$K_S^0 \longrightarrow \pi^+ + \pi^- \qquad (9.2.13)$$

according to the IVB model (9):

$$T(K^0_S \longrightarrow \pi^+ + \pi^-) = 10^{-7}\, m_K\, (5.46 \log_e(m_W/m_p) - 4.68), \quad (9.2.14)$$

and, substituting the experimental value

$$T(K^0_S \longrightarrow \pi^+ + \pi^-) = 7.8 \times 10^{-7}\, m_K \quad (9.2.15)$$

in (9.2.14), we obtain

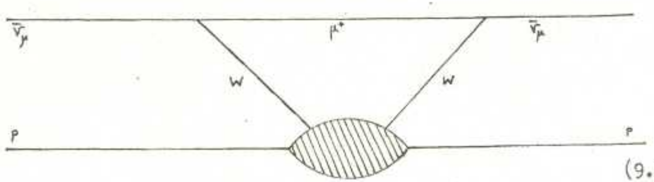
$$m_W \sim 8 \text{ GeV}. \quad (9.2.16)$$

However, it seems very possible that one or more of the approximations used in the derivation of (9.2.14) is unjustified, and hence the result (9.2.16) is inconclusive.

We now examine the effects of the IVB on second-order weak processes. According to the IVB hypothesis, the reaction

$$\bar{\nu}_\mu + p \longrightarrow \bar{\nu}_\mu + p \quad (9.2.17)$$

has a Feynman diagram



(9.2.18)

Using the standard Feynman rules (see Appendix D), we may evaluate the T matrix element for (9.2.18), and, by including an IVB term, we obtain

$$(g_W^2/m_W^2)\,(\Lambda^2/8\pi^2) = (G\,\Lambda^2)/(\sqrt{2}\;8\,\pi^2)\ , \quad (9.2.19)$$

where Λ is the so-called 'weak interaction cut-off energy'. This is the energy at which we cease to integrate over the momenta of particles in loops on Feynman diagrams, and hence avoid infinite and thus physically-meaningless matrix elements. Λ is sometimes identified with the unitarity limit mentioned above, yielding

$$\Lambda \sim 350 \text{ GeV}. \quad (9.2.20)$$

Solving for Λ in (9.2.19), we obtain

$$\Lambda \lesssim 1000 \text{ GeV}.$$

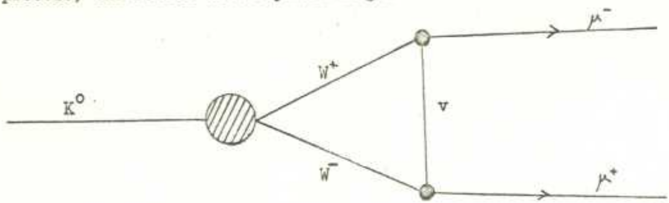
It may be shown that the decay

$$K^0_L \longrightarrow \mu^+ + \mu^- \quad (9.2.21)$$

can occur as a combination of first- and fourth-order electromagnetic interactions. The branching ratio for (9.2.21) through this mechanism has been estimated as (10)

4×10^{-8} . (9.2.22)

However, this decay may also occur as a second- or fourth-order semiweak process, the former with Feynman diagram



(9.2.23)

Once again, evaluating the T matrix element for (9.2.23) and for its fourth-order equivalent, we substitute the experimental branching ratio for the decay (9.2.21):

$\sim 1.4 \times 10^{-6}$, (9.2.24)

and thus obtain

$\Lambda < 75$ GeV . (9.2.25)

However, if

$\Lambda < 10$ GeV , (9.2.26)

then (9.2.21) will proceed primarily as a first-order weak and a fourth-order electromagnetic transition, and hence our information regarding the weak interaction 'cut-off' would vanish. The best method for ascertaining Λ is to calculate the $K_L^o - K_S^o$ mass difference according to the IVB model. Introducing a propagator term into the expression (7.3.41), we finally deduce (11)

$$\Delta m \sim (m_K f_K^2 G_W^2 \sin^2\theta \cos^2\theta \Lambda^2)/(32 \pi^2) , \quad (9.2.27)$$

and substituting the experimental value for m (7.3.38), we predict that

$\Lambda \sim 4$ GeV . (9.2.28)

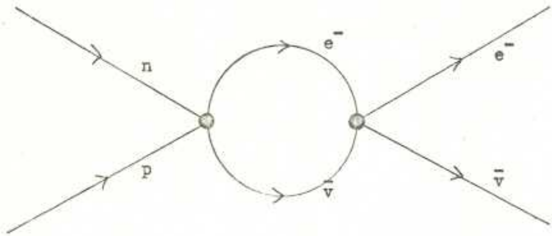
A standard current-current calculation of m yields (12)

$\Lambda \sim 3$ GeV , (9.2.29)

in acceptable agreement with (9.2.28).

There exist a number of further methods of deducing the value of

Λ. The equality of vector coupling constants in neutron and muon decay is initially ensured by the CVC hypothesis. However, virtual weak interactions tend to violate this equality. In neutron decay, we have basically only one type of second-order diagram, assuming that the contribution from virtual baryons is negligible:

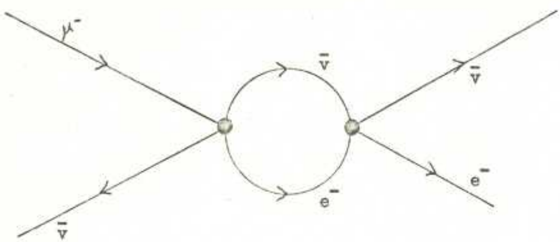


(9.2.30)

Using standard techniques, we find that processes of the type (9.2.30) result in a neutron decay constant of order

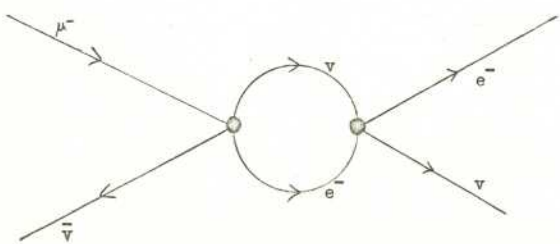
$$(1 + \sqrt{2}\,(\Lambda^2 G)/(2\pi)^2\,)\,G \;. \qquad (9.2.31)$$

In muon decay, there are two distinct kinds of second-order diagram:



(9.2.32)

and



(9.2.33)

We note that the particles in the loop of (9.2.32) have vanishing total

lepton number, while those in the loop of (9.2.33) have $L_{tot} = 2$.
Taking into account processes of the form (9.2.32) and (9.2.33), the muon decay coupling constant now reads

$$(1 + 5\sqrt{2}\ (\Lambda^2 G)/(2\pi)^2)\ G\ . \qquad (9.2.34)$$

Thus we obtain

$$G_n/G_\mu = 1 - 4\sqrt{2}\ (\Lambda^2 G)/(2\pi)^2\ , \qquad (9.2.35)$$

and substituting the experimental result

$$(G_\mu - G_n)/G_n \lesssim 3\%\ , \qquad (9.2.36)$$

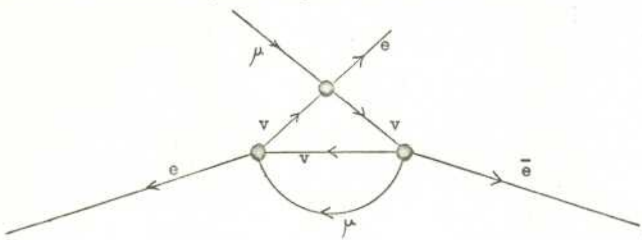
we find that

$$\Lambda < 140\ \text{GeV}\ . \qquad (9.2.37)$$

A more accurate determination of Λ by this method is rendered extremely difficult by the need for electromagnetic corrections in both neutron and muon decay. We now examine two decays which are forbidden by the two neutrino hypothesis, but which, if the latter were invalid, would furnish useful information concerning Λ. The process

$$\mu \longrightarrow e + e^+ + e^- \qquad (9.2.38)$$

has a second-order Feynman diagram of the form



(9.2.39)

The matrix element corresponding to (9.2.39) is equal to

$$(G^2 \Lambda^2)/(2(2\pi)^2)\ (\bar{u}_e \gamma_r (1 + \gamma_5) u_\mu)(\bar{u}_e \gamma_r (1 + \gamma_5) u_e) \qquad (9.2.40)$$

so that its rate is given by (13)

$$W(\mu \longrightarrow e + e^+ + e^-) = 1/(96\ (2\pi)^7)\ G^4 \Lambda^4 m_\mu^5 \qquad (9.2.41)$$

and its branching ratio is

$$(G^2 \Lambda^4)/(2\pi)^4\ . \qquad (9.2.42)$$

The experimental upper limit on the branching ratio of (9.2.38) is (14)

1.5×10^{-7} , (9.2.43)

and substituting this value in (9.2.42), we obtain

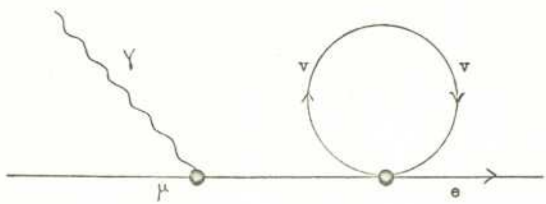
$\Lambda < 35$ GeV . (9.2.44)

If the neutrino loop is found to give zero contribution (15), then (9.2.38) must occur to third-order in the weak interaction, setting an upper limit of a few hundred GeV on .

Finally, we examine the 'forbiden' process

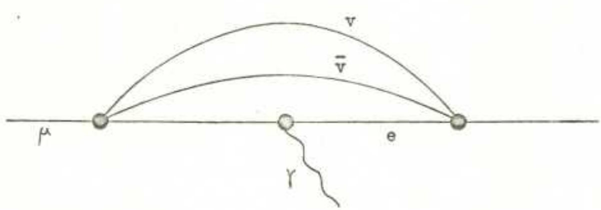
$\mu \longrightarrow e + \gamma$. (9.2.45)

In first-order, this might occur as



(9.2.46)

but it may be shown that the matrix element corresponding to (9.2.46) and associated diagrams vanishes identically because of the (V-A) theory. However, second-order graphs of the type



(9.2.47)

do produce non-vanishing coupling constants of order

$e\, G^2 \Lambda^2 \log_e(\Lambda/m_\mu)^2$. 9.2.48)

From (9.2.48), we may calculate that the branching ratio for (9.2.47) and similar diagrams is given by

$R = (8/3\pi^4)\ (e^2/4\pi)\ G^2 \Lambda^4 (\log_e(\Lambda/m_\mu)^2)^2$, (9.2.49)

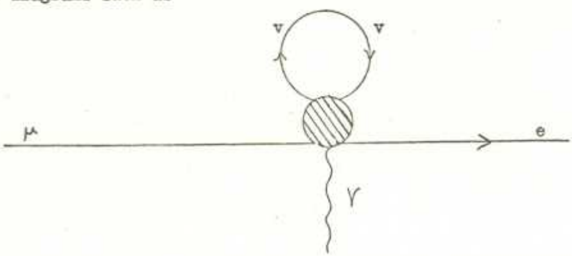
where e is the universal electromagnetic coupling constant. Since experiments give

$$R \leqslant 6 \times 10^{-9}, \qquad (9.2.50)$$

substitution in (9.2.49) yields

$$\Lambda < 10 \text{ GeV}. \qquad (9.2.51)$$

The result (9.2.51) might suggest that if we are to assign a small enough value to Λ, then the decay (9.2.45) might have a very small branching ratio regardless of any electron-muon conservation laws. However, if we assume the weak interaction to be of a non-local nature, then diagrams such as



(9.2.52)

must be included in the total branching ratio for (9.2.45). In general, it is impossible to calculate the contribution from graphs of the form (9.2.52), but in this case, assuming that the non-locality of the weak interaction results from the existence of a massive charged IVB, the calculation is rendered possible. It has been demonstrated that the rate for (9.2.52) is given by (16)

$$R \sim (3e/8\pi)\ f(M/\Lambda), \qquad (9.2.53)$$

where f is a form factor and M is the IVB mass, so long as the magnetic moment of the W particle is precisely unity, as predicted by the Dirac equation[4]. The form factor in (9.2.53) is such that, for

$$\Lambda \gg M, \qquad (9.2.54)$$

$$f(M/\Lambda) \simeq \log_e(\Lambda^2/M^2)^2, \qquad (9.2.55)$$

producing an unphysical result. If

$$\Lambda \sim M, \qquad (9.2.56)$$

then we obtain

$$R \sim 10^{-4}, \qquad (9.2.57)$$

which is more satisfactory than (9.2.55), but still not correct. If, however, we assign non-unit magnetic moment to the IVB, then at

$$\mu_{IVB} \sim 1.7 \text{ n.m.}, \tag{9.2.58}$$

R becomes vanishing small. The presence of an 'anomalous magnetic moment' or G-factor over and above the Dirac prediction of one, is thought to be due to the strong interaction. However, as we shall see, the IVB does not take part in strong interactions, and hence it appears unlikely that it possesses a nonzero anomalous magnetic moment.

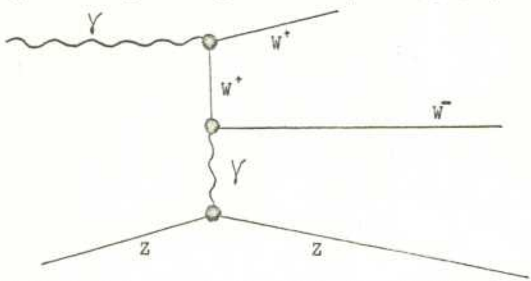
9.3 The Production and Decay of the W Particle.

The charged IVB which we have considered above may be produced either by the electromagnetic interaction on its own, by a combination of strong and semiweak interactions, or by the electromagnetic and semiweak interactions. Electromagnetically, the W particle can be produced in either of the two reactions:

$$\gamma + Z \longrightarrow Z + W^+ + W^-, \tag{9.3.1}$$

$$e^+ + e^- \longrightarrow W^+ + W^-. \tag{9.3.2}$$

A possible Feynman diagram for the process (9.3.1) is



(9.3.3)

At low energies, scattering of the type (9.3.3) will be predominantly coherent[5], but at higher energies, incoherent production will begin to contribute significantly. We find that we may express the total cross-section for all processes (9.3.1) as (17)

$$\sigma_{tot} = \max\left(\sigma_{coh},\ Z\sigma_p + (1 - 1/Z)\,\sigma_{coh}\right), \tag{9.3.4}$$

where the second term in the bracket is an approximate expression for the total cross-section when incoherent scattering becomes important (σ_p denotes the production cross-section on a proton). If, for example, the target nucleus is iron ($Z = 26$), and the W is assumed to have unit magnetic moment and a mass of 2 GeV, then we obtain the following values for σ_{tot} with gamma rays of varying energy: (18)

E_γ (lab.) (GeV)	(1/26) σ_{tot} (cm^2)
6	8.4×10^{-44}
8	1.4×10^{-42}
10	9.6×10^{-42}
15	1.9×10^{-37}
20	1.6×10^{-36}

(9.3.5)

The possible production reaction (9.3.2) will be discussed in Chapter 10.

According to the Hamiltonian (9.1.13), the W particle is coupled to the hadronic current via the semiweak interaction. Thus the IVB may be produced in any reaction of the form

$$A \longrightarrow B + W^{\pm}, \qquad (9.3.6)$$

where A and B are any two hadron states coupled by the hadron current. The most experimentally-viable processes of the type (9.3.6) are

$$\pi^{\pm} + p \longrightarrow p + W^{\pm}, \qquad (9.3.7)$$

$$K^{\pm} + p \longrightarrow p + W^{\pm}, \qquad (9.3.8)$$

$$p + p \longrightarrow d + W^{\pm}. \qquad (9.3.9)$$

In (9.3.7), it has been shown that the total cross-section for W production is given by (19)

$$\sigma_{tot} = 10^{-32} \times F_{\pi}^2(-m_W^2) \quad cm^2 , \qquad (9.3.10)$$

where F is the pion form factor. Assuming that F is dominated by the ρ meson pole, we find that

$$F_{\pi}(-m_W^2) \sim -1/6 , \qquad (9.3.11)$$

and substitution of the result (9.3.11) in (9.3.10) yields

$$\sigma_{tot} \sim 3 \times 10^{-34} \ cm^2 . \qquad (9.3.12)$$

Since (9.3.8) involves the $\Delta Y = 1$ rather than the $\Delta Y = 0$ hadron current, its cross-section must be less than that for (9.3.7) by a factor of $\tan^2\theta$, where θ is the Cabibbo angle (see 5.6 and 8.3). Experimentally, if the pion decay mode of the IVB were dominant, then the reactions (9.3.7) and (9.3.8) would appear as p   p   n ,   (9.3.13)

K   p   p   (n),   (9.3.14)

so that, whereas (9.3.13) would pass unobserved because of the many similar strong interaction processes, (9.3.14) would be conspicuous because it does not conserve strangeness. In the case of (9.3.9), it has been calculated (20) that the cross-section for W production should be

$$10^{-33} \quad cm^2 , \qquad (9.3.15)$$

assuming an IVB mass of 2 GeV, but this is probably too large, since it was calculated in analogy to the process

p   p   d   ,   (9.3.16)

where the momentum transfer is much smaller.

Finally, we discuss IVB production via the electromagnetic and semiweak interactions. This may occur through any of the reactions

(K)   p   p   W ,   (9.3.17)

p   n   W ,   (9.3.18)

v   Z   Z   W ,   (9.3.19)

where the process (9.3.17) occurs via photon exchange instead of via pion exchange as above. It has a Feynman diagram

(9.3.20)

It may be shown (21) that the differential cross-section for (9.3.20) has a maximum

$$d \quad /dq^2 \qquad 4 \times 10^{-35} \; cm^2/(GeV)^2 \qquad (9.3.21)$$

for IVB mass 2 GeV and energy 4 GeV in the c.m.s. Using dispersion relations we find (22) that the cross-section for (9.3.18) is