NUKLEONIKA — VOL. 26 — No. 2/81

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WEAK DECAYS*) **)

Weak decays of strange, charmed and heavier mesons are discussed in the context of QCD.

Introduction

The short range $\sim 1/m_W \sim 10^{-3}$ fm of W exchanges compared to the distances ~ 1 fm at which QCD interactions become strong inspires the hope that QCD perturbation theory may be relevant even in low-energy hadronic weak processes. As usual, however, the development of initial or final hadron states at large distances, where quarks and gluons are nearly on their mass shells, is quite inaccessible by perturbation theory. Only in inclusive measurements on very high invariant mass hadron systems are these effects unimportant. In strange particle decays and to a large extent also τ lepton and charmed hadron decays the evolution of the hadron final state at large distances is crucial, and QCD perturbation theory is unable to provide a precise quantitative description: it must be supplemented by estimates of hadronic effects based at best qualitatively on QCD. In inclusive weak decays of heavier (e.g., b) hadrons, the final state hadronic effects should be less important; however, when the decaying hadron contains light quarks, the structure of the initial state cannot be estimated reliably. In applications of QCD to processes involving large momentum transfers (e.g., deep inelastic γ^*N scattering), ignorance of initial hadron structure may largely be overcome by comparison of results for different values of the large momentum, in which small momentum details are factorized out: this procedure is usually inapplicable in weak decays since the initial hadron is fixed. In the face of these severe difficulties, I do not attempt a precise quantitative analysis of weak decays; rather, I shall consider the basic physical phenomena which appear to be dominant in weak decays, with the hope that their qualitative effects will remain unchanged in an eventual exact treatment. This paper is a review in that it covers a very broad area; many novel points are, however, discussed.

I discuss the several varieties of weak hadronic decays in turn. The simplest is probably the decay of a real W boson: if all quark masses are ignored (the chiral symmetry limit),

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*) Presented at the Third Warsaw Symposium on Elementary Particle Physics, Jodłowy Dwór, Poland May 22–28, 1980.

**) Work supported in part by the U.S. Department of Energy under Contract No. DE-AC-03-79ER0068 and by Fleischmann Foundation Grant No. 30175.

inclusive properties of the hadron final state are the same as would result from decay of a virtual photon (γ^*) with invariant mass $Q \simeq M_W$. This simplicity is preserved when virtual W bosons (W^*) are produced by a purely leptonic process, such as in heavy lepton decay $\tau \to \nu W^*$. The small τ mass renders neglect of hadronic effects in W^* decay impossible: nevertheless, the total decay rate is (fortuitously) well approximated by perturbation theory. Decays $\tau \to \nu_\tau M$ to single mesons M give information on W^*M couplings. The same couplings appear in the weak decays $M \to W^* \to l\nu_l$ of these mesons: for pseudoscalar M they measure the violation of chiral symmetry in QCD. Ignorance of M structure prevents direct estimation of such decays, except for cases such as $T \to \tau\bar{\tau}$. After these leptonic decays, the next major class of weak decays discussed below are the semileptonic ones, of the form $H \to XW^*_{\to l\nu}$, where H is a hadron (meson or baryon) and X denotes any hadronic system. Most such decays proceed through a decay $Q \to q'W^*_{\to l\bar{\nu}_l}$ of the heavy quark in H (although in some meson decays, processes such as $Q\bar{q} \to l\bar{\nu}_l X$, where $\bar{q}$ is a "spectator" quark in H, may be important, as discussed below). Final state gluon emission and interactions with "spectator" quarks from the initial H may modify the rate for the complete decay: for charmed meson decays, there is some phenomenological evidence that the modifications to the total rate are small; the lepton energy spectrum is, however, considerably softened. For strange particle decays, the small number of accessible final states make coherent hadronic effects crucial: nevertheless, rates for decays to specific final states may often be estimated by symmetry considerations. The most complicated but most revealing weak decays are the non-leptonic ones. The dominant fundamental mechanisms for such decays are probably (a) $Q \to q\, q\bar{q}$, (*b*) $Q\bar{q} \to q\bar{q}$ (or $Qq \to qq$) and (c) $Q \to qG^*$ (q are light quarks and G denotes a gluon). (Process (a) occurs by diagrams analogous to μ decay; (b) occurs through s or t, or channel W^* exchange, depending on the initial $\bar{q}$ charge; (c) occurs through a W^* loop, yielding a (color) charge radius term). In strange particle decays, all three mechanisms may be present. Their relative importance is however probably determined not so much by their intrinsic rates as by the structure of the decaying strange hadrons. A striking phenomenological observation is that $|\Delta I| = 1/2$ non-leptonic strange particle decays are enhanced by a factor of $\simeq 20$ in amplitude over $|\Delta I| = 3/2$ ones, while simple free quark estimates (which necessarily ignore process (c) since it involves a gluon) suggest that they should be comparable. This phenomenon might well be explained if the process (c) dominated these decays (since $s \to dG$ is pure $|\Delta I| = 1/2$): however, no reliable quantitative estimates of its importance are available, but there are some qualitative suggestions that it is insufficient. The isospin structure in processes (a) and (b) depends on the color symmetries of their initial and final states. In the free quark approximation, the $|\Delta I| = 1/2$ and $|\Delta I| = 3/2$ components are of equal strength, but there are indications that gluon exchanges tend to enhance the rates for color channels which give $|\Delta I| = 1/2$: hadronic effects at large distances may also effect such an enhancement (as suggested by soft pion results in specific cases). In charmed particle decays, measurements of semileptonic branching ratios imply a much smaller enhancement of nonleptonic modes than in strange particle decays. The process (c) is Cabibbo suppressed for c decays, since it leads to $S = 0$ final states; the phenomenological suppression of such final states suggests that (c) is indeed unimportant in this case. The observation of a larger hadronic decay width for D^0 than D^+ suggests that process (b) may dominate over (a) in the former case, and be unimportant by virtue of Cabibbo suppression in the latter, leaving (a) dominant. Quantitative estimates of this effect are however difficult. The structure of the decays of b and heavier quarks are determined by the forms of their weak couplings. Non-leptonic decays of pseudoscalar mesons containing heavy quarks are probably increasingly dominated by process (a). The presence of light quarks in the lowest-lying such pseudoscalar mesons precludes accurate estimates of their structure. However, weak decays of mesons containing predominantly heavy quarks should be more amenable to reliable estimation: For example, weak parity-violating admixtures into inclusive non-leptonic Y decays $((b\bar{b}) \to q\bar{q})$ may be calculated with some certainty, and are perhaps not beyond experimental reach at the 0 (1%) level. The possibility of proton decay via $qq \to \bar{l}\bar{q}$ in grand unified gauge models has recently received much attention. Modifications to the free quark approximation for this decay rate may occur just as they do for ordinary weak decays; gluon exchange effects may be more important because of the shorter range of the primary interaction (and thus the larger the possible exchanged gluon momenta); nevertheless mundane hadronic effects probably introduce larger uncertainties.

The Weak Currents

The W apparently couples to the weak charged current

$$J^w_\mu = (\bar{u}_L, \bar{c}_L, \bar{t}_L, \dots)\gamma_\mu U \begin{pmatrix} d_L \\ s_L \\ b_L \\ \vdots \end{pmatrix} + (\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau, \dots)\gamma_\mu \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \\ \vdots \end{pmatrix},$$

where the suffix L denotes the left-handed $((1-\gamma_5)/2)$ projections of fermion fields, and U is a unitary matrix of mixing angles which connects the quark fields participating in weak interactions to the mass eigenstates.

(In the u, d, s, c sector, U is the Cabibbo rotation matrix $\begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$ with $\theta_c \simeq 0.2$; on including b, t, U may contain CP-violating phases. Approximating for now U by the identity matrix, the Lorentz vector $(\bar{q}\,\gamma_\mu q)$ part of (1) is related to the isovector part of the electromagnetic current by a weak isospin rotation $(u \leftrightarrow d, s \leftrightarrow c, \dots)$. Differences between vector current W^* "decays" and isovector γ^* "decays" arise from violations of weak isospin invariance by mass differences between pairs of quarks (and leptons) in the same weak isomultiplet (e.g., u, d; c, s). If quark masses are introduced explicitly into the original Lagrangian, then the rate for $\gamma^* \to q\bar{q}$ at high Q in the free quark approximation $\simeq 1-2\,(m/Q)^4$, while the rate for vector current decays $W^* \to q_1\bar{q}_2$ is $\simeq 1-$ $-[(m_1-m_2)/Q]^2$ (Ref. [1]). The electromagnetic current is exactly conserved (so that the spin-0 components of γ^* are decoupled): this conservation (CVC) for the weak vector current holds only in the weak isospin symmetric limit. The weak vector current in general has a divergence $\sim (m_1-m_2)$ [F1], so that spin-0 W^* may decay through the vector current at a rate $\sim [(m_1-m_2)/Q]^2$. In the chiral symmetry limit where all quark masses are neglected, axial vector current W decays should become identical to vector current ones: for finite m/Q, the axial current W^* decay rate $\simeq 1+m_1 m_2/Q^2$. The divergence of the axial current $\sim (m_1+m_2)$ (so that spin-0 W^* decay through the axial current at a rate $\simeq [(m_1+m_2/Q]^2)$, which is non-zero whenever finite quark masses are present to break chiral symmetry. The large masses of s, c, ... quarks presumably arise primarily from weak interaction effects (via the Higgs mechanism, etc.), and must be inserted as explicit bare mass terms in the QCD Lagrangian. However, the confinement of quarks into color singlets with radii $\leqslant 1/\Lambda$ presumably contributes additional dynamical effective quark masses $\sim \Lambda$ (which decrease $\sim \Lambda^3/Q^2$ at short distances) independent of flavor. These contributions are important for the divergence of the u, d axial current, but tend to cancel in the divergence of the u, d vector current: the validity of CVC results is thus presumably a consequence of the fact that most hadronic interactions involve $Q \sim \Lambda$, and $|m_u - m_d|/\Lambda \ll 1$.

Weak interactions occur between the currents (1) by W^* exchanges. In the low-energy limit, and ignoring QED and QCD corrections (free quark approximation), the interactions are represented by an effective four-fermion vertex $H_W^{eff} \simeq \frac{G_F}{\sqrt{2}} \{J_\mu^W, (J_\mu^W)^+\}$ $(G_F/\sqrt{2} = g^2/(4M_W^2))$. As discussed below, QCD corrections to these interactions are sensitive to their short distance structure, so that their softening at $Q \geqslant M_W$ must be accounted for. QED corrections are in some cases also sensitive to the possibility of $WW\gamma$ couplings, which must be treated in the complete Weinberg-Salam model. In these notes, I shall not consider higher-order weak processes, as presumably induce $K_L^0 \to \gamma\gamma$, $K_L^0 \to \mu\bar{\mu}$, $m_{K_L} - m_{K_S} \neq 0$ etc. I shall also not discuss CP-violating effects.

The mixing matrix U appears to be arranged so that $\Gamma(c \to sW^*) \gg \Gamma(c \to dW^*)$, $\Gamma(b \to cW^*) \gg \Gamma(b \to uW^*)$ and presumably $\Gamma(t \to bW^*) \gg \Gamma(t \to sW^*) \gg \Gamma(t \to dW^*)$. I shall assume this ordering below.

W^* Decays

The decay of an "isolated" W^* with large invariant mass Q is similar to that of a virtual photon (γ^*) with the same mass; the latter case is discussed in Ref. [2]. The W^* decay is initiated by the production of a $q_1\bar{q}_2$ pair. The possibility of final state interactions allows these quarks to be produced with a spectrum of invariant masses extending up to the kinematic limit $\simeq Q$. Very large quark invariant masses are dissipated by radiation of a cascade of gluons at short distances, as described by perturbation theory. However, for parton invariant masses below a critical $\mu_c = 0(\Lambda) = 0(1 \text{ GeV})$, this perturbative description becomes inadequate, and one must resort to a largely phenomenological model for the final formation of hadrons.

At present, the only available source of "isolated" W^* is heavy leptons decay $\tau \to \nu_\tau W^*$, in which the W^* has an invariant mass spectrum $1/\Gamma \, d\Gamma/dQ \simeq 2[1+2(Q/m_\tau)^2] [1-(Q/m_\tau)^2]^2$ (with $Q \leqslant m_\tau$, and assuming the observed $V-A$ $\tau\nu_\tau W$ coupling): the decays of W^* thus produced are therefore dominated by the region of low parton invariant masses, inaccessible to perturbation theory. In the free quark approximation (presumably valid as $Q^2 \to \infty$) the diagram of Fig. 1 for τ decay implies a leptonic branching ratio $\simeq 2/5$ (recall the three possible colors of a quark pair). Perturbative gluon exchanges between the final quarks modify this to $\simeq 2/[5(1+\alpha_s(m_\tau^2)/\pi)]$ [F2]. Ignoring quark masses, the complete leptonic branching ratio may be estimated using the relation of weak and electromagnetic currents mentioned above by integration of the observed isovector γ^* decay

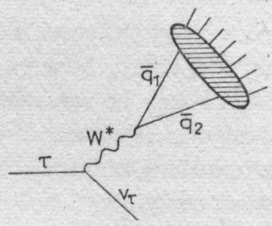

Fig. 1. Schematic diagram for semileptonic decay of a τ heavy lepton.

rate (this isospin component is identified by even numbers of final π) over the W^* invariant mass spectrum produced in τ decay, suggesting an $\sim 18\%$ suppression relative to the free quark result [3]. (For $\Lambda \simeq 0.5$ GeV, this estimate fortuitously agrees with the perturbative result. Experimental measurements are as yet of insufficient precision to test the estimate (e.g., Ref. [4]).

In addition to their total rate, other inclusive features of τ decays may be considered. The angular distribution of final state hadronic energy could be calculated at high Q^2 by QCD perturbation theory, but in τ decays is dominated by hadronic effects, leaving no trace of two quark jets. For any serious application of QCD perturbation theory to be possible, it is essential that the final formation of hadrons is sensitive only to the local structure of the parton state, and not on its global properties or details of its production (for further discussion on this point, see Ref. [2]). The approximate agreement between multiplicities measured in τ decays and in related γ^* decays supports this belief.

Purely Leptonic Decays

I now consider direct couplings of mesons M to virtual W^*, as illustrated in Fig. 2. These couplings enter in the leptonic decays $M \to W^* \to l\bar{\nu}_l$ and in exclusive heavy lepton decays $\tau \to \nu_\tau W^*_{\to M}$. Couplings of W^* to vector mesons are related by (weak) isospin rotations to the corresponding γ^* couplings, which may be measured in $M \to \gamma^* \to l\bar{l}$. The coupling of Fig. 2 is proportional to the amplitude for the (valence) $q\bar{q}$ in M to an-

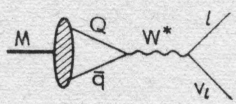

Fig. 2. Schematic diagram for purely leptonic decay of a meson M.

nihilate "at a point" into W^*, and hence to the $q\bar{q}$ "wavefunction at the origin" $\psi(0)$. The complete coupling should presumably involve the product of this amplitude with the amplitude for the other constituents (or bag) of the M state to disappear (into the "vacuum"). A possible rationale for neglect of this latter term might be that without the "valence" $q\bar{q}$, the contents of the meson are indistinguishable from fluctuations which would occur in the "vacuum" regardless of the presence of the meson [5]. For mesons in which the q, $\bar{q}$ masses are sufficiently large compared to Λ, the q, $\bar{q}$ should be non-relativistic and nearly on-mass shell (so that admixtures of e.g., $q\bar{q}G$ states into the M wavefunction are negligible): in this case a non-relativistic $\psi(0)$ may be defined and estimated using the Schrödinger equation from a potential (or bag) model. This treatment is probably suitable for heavy $Q\bar{Q}$ resonances (e.g., ψ, Y; denoted generically ζ throughout). (At high ζ masses, interference between $\zeta \to Z^* \to \tau\bar{\tau}$ and the dominant leptonic decay mechanism $\zeta \to \gamma^* \to \tau\bar{\tau}$ may be revealed by τ longitudinal polarizations measured through decay product angular asymmetries [6]. Mesons such as $(b\bar{c})$ will probably not have significant leptonic branching ratios, since their lowest-lying states will presumably be pseudoscalar). For mesons containing any light quarks, this approach probably fails. When constituent particles may be off their mass shells (as in any relativistic formulation), the meaning of $\psi(0)$ becomes unclear: the relevant integral of the full Bethe-Salpeter wavefunctions depends on the invariant masses of the annihilating particles. In addition, the large size of a meson containing light quarks precludes any reliable estimate of a gluon exchange potential.

In most cases involving light quarks, Fig. 2 may be treated only by phenomenological means. The coupling $W^*\varrho$ may be deduced from $\gamma^*\varrho$ by an isospin rotation; the result agrees with the measured $\tau \to \varrho\nu_\tau$ decay rate. There is now also reasonable experimental evidence for a resonant decay $\tau \to \mathrm{Al}\nu_\tau$[7]; its rate is consistent with a W^*Al coupling $\sim 1/\sqrt{2}$ the $W^*\varrho$ coupling, as expected in the chiral symmetry limit. On the other hand, in the non-relativistic limit, $|\psi(0)|^2$ vanishes for the Al, since it is a p-wave $q\bar{q}$ state. The experimental absence of such a suppression suggests that this is not a relevant limit: in a relativistic Al state, the "lower components" of the q, $\bar{q}$ Dirac spinors (which essentially correspond to an s-wave state [F3], become important, leaving no trace of the nominal p-wave assignment in the chiral symmetric limit $m_q \to 0$. Note that W^* couplings to $J^{PC} = 1^{+-}$ mesons, such as $B(1235)$, would be "second class" (proportional to $q_\nu \sigma_{\mu\nu}\gamma_5$), and may occur only at the level of isospin violations [8] so that the rate for e.g., $\tau \to B(1235)\nu_\tau$ should be very small (perhaps $\simeq 10^{-3}$ times the $\tau \to \mathrm{Al}\,\nu_\tau$ rate).

Pseudoscalar mesons may couple to spin-0 W^* through the divergence of the axial vector current. The rate for their resulting leptonic decays is given by $\Gamma(M \to W^* \to l\nu) \simeq G_F^2 f_M^2 m_M^3 (m_l/m_M)^2/(8\pi)$ $(m_l \ll m_M)$, where f_M parametrizes the MW^* coupling. The factor $(m_l/m_M)^2$ in this rate represents a "helicity suppression" which arises because the W^* couples to the left-handed l, but the spin of the original M constrains the total angular momentum of the final state to be zero, thus requiring that the helicity of the l be opposite to its spin, and introducing a factor m_l/E_l into the decay amplitude. This factor renders leptonic decays miniscule for $m_M \gg m_l$. Helicity suppression also affects the initial $q\bar{q} \to W^*$ annihilation: the W^* must couple to left-handed q, $\bar{q}$, but the total M state has spin-0. This introduces a helicity suppression factor $\sim (m_q + m_{\bar{q}})/(E_q + E_{\bar{q}}) \sim (m_q + m_{\bar{q}})/m_M$ just as in the divergence of the (free quark [F4]) axial vector current $\partial_\mu \bar{q}\gamma_\mu\gamma_5 q \simeq (m_q + m_q)\bar{q}\gamma_5 q$: when $m_{q,\bar{q}} \to 0$, the axial current is conserved and the M leptonic decay rate vanishes. The complete W^*M coupling is the product of the helicity suppression factor and $|\psi(0)|^2$ (which gives roughly the inverse volume of the $q\bar{q}$ state). Typically, the value of $|\psi(0)|^2$ depends on the reduced mass of the q, $\bar{q}$ [F5], suggesting that all mesons containing light quarks should have similar $|\psi(0)|^2$ (irrespective of the heavy quark mass). (The similarity of the wavefunctions for π, K is supported by the similarity of their electromagnetic charge radii). In any non-relativistic treatment, the annihilation helicity suppression factor differs from one only by terms $0(E_{BE}/m_M)$ where E_{BE} is the M binding energy, which cannot consistently be kept. If the mass of M is dominated by the rest mass of one of its constituent quarks (as for D mesons), then no significant helicity suppression should occur. Ignoring helicity suppression for π, K and taking $|\psi_K(0)|^2 \simeq |\psi_\pi(0)|^2$ suggests $f_K \simeq f_\pi \sqrt{m_\pi/m_K}$ (Ref. [10]), where $f_M^2 \simeq 2|\psi_M(0)|^2/m_M$. Experimentally, $f_K \simeq f_\pi (\simeq m_\pi)$. Introducing a $(m_q+m_{\bar{q}})/m_M$ suppression factor into f_M gives $f_K \simeq f_\pi[(m_s+m_u)/(m_u+ +m_d)]\ [m_\pi/m_K]^{3/2}$ if $|\psi_K(0)|^2 = |\psi_\pi(0)|^2$. Taking $|\psi_\varrho(0)|^2$ from the ϱ charge radius gives roughly the correct γ_ϱ for the helicity unsuppressed coupling $\varrho \to \gamma^*$. Using the π, K charge radii to estimate $|\psi_{\pi,K}(0)|^2 \sim [m_{\varrho,K^*}]^3$ yields too large a value for $f_{\pi,K}$: this suggests either that the helicity suppression is numerically important, or that the approximate relation between size and "wavefunction at origin" fails for the ultrarelativistic π, K states. Admission of the ultrarelativistic nature of the K, π could well allow $|\psi_K(0)|^2 \neq |\psi_\pi(0)|^2$: f_M is determined by the difference between the Bethe-Salpeter wavefunctions corresponding to the upper components of q, $\bar{q}$ Dirac spinors and their lower components [F6] (Ref. [11] which might, for example, be proportional to the larger quark mass. For heavy pseudoscalar mesons, the rates for purely leptonic decays to e, μ are probably rendered negligible by the $(m_l/m_M)^2$ helicity suppression factor. However, decays to τ leptons may well be significant. For charmed mesons, only the decay $F \to \tau\nu_\tau$ escapes Cabibbo suppression, while for b-mesons, mixing angles probably suppress all but $B_c \equiv (b\bar{c}) \to \tau\nu_\tau$. Since these two states contain no (valence) u, d quarks, non-relativistic estimates of their wavefunctions are plausibly adequate: taking a logarithmic interquark potential yields $f_F \simeq 170$ MeV, $f_{B_c} \simeq 520$ MeV, while a linear potential gives $f_F \simeq 230$ MeV, $f_{B_c} \simeq 650$ MeV [F7]. To deduce the corresponding leptonic branching ratios, one must estimate the total non-leptonic decay rates as discussed below. $\tau\nu_\tau$ decays may provide useful signatures for heavy meson production [12]. The estimation of f_D involves similar relativistic complications as for $f_{\pi,K}$. One possible (but dubious) approach proceeds as follows: the behavior of the spin-0 axial vector spectral function (spin-0 W^* decay rate through the axial vector current) may be estimated from QCD perturbation at high Q^2. This estimate may provide a finite energy sum rule for the integral of the actual spectral function, which certainly does not follow the perturbative form at low Q^2. Then the actual spectral function is approximated by a single $f_M\delta(Q^2-m_M^2)$, and the resulting integral compared with the perturbative estimate in the finite energy sum rules. A direct application of this procedure yields $f_D \simeq 1/(4\sqrt{2}\pi)\ m_D^2/m_c \simeq 150$ MeV $\simeq f_F$ [13]. It is very difficult to make a serious estimate of the errors in this result, but they are probably at least $0(\Lambda) \sim 0.5$ GeV. This and the non-relativistic estimates given above suggest that in very heavy pseudoscalar mesons f_M will not rise sufficiently to counteract the larger helicity suppressions in leptonic decays, so that their branching will become negligible. One possible circumstance in which this phenomenon is evaded would occur if the lowest-lying meson carrying a new flavor had $J = 1$ rather than $J = 0$ (this appears likely only if the new quark had $J = 3/2$): in that case $l\bar{\nu}_l$ decays would be significant. Note, however, that MW^* couplings could be measured in $L \to M\nu_L$ without helicity suppressions if a suitably placed heavy lepton L exists. (Similar information could perhaps also be extracted from diffractive M neutrino production $\nu N \to \mu MX$). It is also possible that at high Q^2, the process $e^+e^- \to Z^\circ \to M$ (where M is a pseudoscalar heavy meson) may occur through the divergence of the axial vector Z° current: its presence would be revealed by isotropic angular distributions of M decay products with respect to the original $e^\pm$ direction.

One effect which may in part overcome helicity suppression of leptonic pseudoscalar meson decays is emission of hard photons from the incoming M. In the discussion of hadronic decays below, we will encounter similar considerations for gluon emission. The vector nature of the photon coupling prevents soft photon radiation from changing the spin of a state: however, hard photon emissions from an initial M may carry away angular momentum, leaving effectively a spin-1 "meson" state, whose leptonic decays suffer no helicity suppression [F8]. If no photon emission occurs, then the leptonic decay rate for a pseudoscalar meson is given by $\Gamma_0 \equiv \Gamma(M \to l\nu_l) \simeq G_F^2 f_M^2 m_M^3 (m_l/m_M)^2/(8\pi)$ $(m_l \ll m_M)$, where quark mixing angles are absorbed into G_F. Soft photon emissions yield $\Gamma(m \to \to l\nu_l) + \Gamma(M \to l\nu_l\gamma) \simeq (1-3\alpha/\pi \log(m_M/m_l))\Gamma_0$ (this correction exponentiates when summed to all orders in $\alpha \log(m_M/m_\pi)$). Hard photon ("structure dependent") radiation potentially provides a much larger correction, since it can overcome the $0(m_l/m_M)$ helicity suppresion. In practice, $\Gamma_{SD}(\pi \to e\nu\gamma)/\Gamma(\pi \to e\nu) \sim 10^{-3}$ [Ref. 15], while $\Gamma_{SD}(K \to \to e\nu\gamma)/\Gamma(K \to e\nu) \simeq 1.05 \pm 0.25$ (Ref. [16]). For $\pi \to e\nu\gamma$, the vector current contribution may be estimated using CVC from $\pi^\circ \to \gamma\gamma$ (although if $(m_u - m_d)/(m_u + m_d)$ is large there may be significant deviations from CVC [17]). In other cases, the rates are estimated by phenomenological Lagrangians involving intermediate ϱ, Al, K^*, ... fields. One may also attempt a static quark approximation [14]. The amplitude for a photon to reverse the spin of a quark in M is proportional to its effective magnetic moment $\mu_q \sim e/m_q$ $(m_q \gg \Lambda)$, $\sim e/\Lambda(m_q \lesssim \Lambda)$ [F9]. Only photons emitted incoherently from the q, $\bar{q}$ in M may change the total M spin. A naive calculation in the free static quark approximation then suggests $\Gamma_{SD}(M \to l\nu_l\gamma) \simeq G_F^2 f_M^2 m_M^5 (\mu_q - \mu_{\bar{q}})^2/(7680\pi^3) \sim G_F^2 f_M^2 m_M^7 \alpha/(1920\pi^2$ $(m_q^{eff} m_{\bar{q}}^{eff})^2)$: taking $m_q^{eff} \sim \Lambda \sim 0.5$ GeV, this overestimates $\Gamma(\pi \to e\nu\gamma)$ and $\Gamma(K \to e\nu\gamma)$ by about a factor ten. (If instead $m_q^{eff} \sim m_\pi, m_K$, a gross overestimate results). The decay $D \to e\nu\gamma$ is Cabibbo suppressed; $F \to e\nu\gamma$ is not, but probably has a branching ratio $\sim 10^{-5}$. For heavy $(Q\bar{q})$ mesons probably $|\psi(0)|^2 \sim [m_{\bar{q}}^{eff}]^3$: thus $\Gamma_{SD}(M \to l\nu_l\gamma) \sim \sim G_F^2 m_M^5 (10^{-4}\alpha)$ $(m_{\bar{q}}^{eff}/m_M)$, which becomes relatively smaller as $m_Q \to \infty$.

Semileptonic Decays

Having discussed purely leptonic decays, I now turn to semileptonic decays of the form $M \to XW^*_{\to l\bar{\nu}_l}$, where X denotes any hadronic system, and M is a pseudoscalar meson containing a heavy (unstable) quark Q and a lighter antiquark $\bar{q}$ (e.g., $M = K, D, F, \ldots$). Such decays are of particular significance because the leptons they produce are in many reactions the only signals for heavy flavor production. The basic diagrams for semileptonic M decays are shown schematically in Fig. 3. In Fig. 3(a), the heavy quark Q undergoes an independent semileptonic decay, while in Fig. 3(b), it disappears through annihilation with a "spectator" antiquark in the initial M into a $l\bar{\nu}_l$ pair: in most cases, the mechanism (a) probably dominates. The rough size of the incoming M state is $\sim 1/\Lambda$ if $m_{\bar{q}} \simeq 0$ (or $\sim 1/m_q$ otherwise): hence its constituents are typically off-shell by an amount $\sim \Lambda$, giving

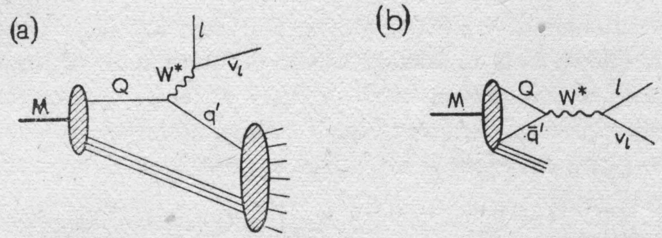

Fig. 3. Schematic diagrams for semileptonic decay of a $(Q\bar{q})$ meson M.

the Q, a Fermi momentum $\sim \Lambda$. So long as $m_Q \gg \Lambda$ (as for c, b), it should probably be adequate to estimate the process of Fig. 3(a) by approximating the initial Q as free. For sufficiently large m_Q, inclusive M decays in which all possible final hadron systems X are summed over, may be estimated to $0(\alpha_s^0)$ by treating the final q' as free. In this approximation (and for simplicity taking at first $m_{q'} = 0$), the l energy spectrum is analogous to the e^- spectrum in μ decay, and is given by $1/\Gamma_0 d\Gamma_0/dx = 2x^2(3-2x)$ (for negatively-charged Q) or $12x^2(1-x)$ (for positively-charged Q), where $x = 2E_l/m_Q \lesssim (1-m_{q'}^2/m_Q^2)$ [F10]. Thus $\langle E_l/m_Q\rangle = \langle x/2\rangle \simeq 0.3$ or 0.35 (note that $\langle E_l/E_Q\rangle$ is conveniently a Lorentz invariant for massless l and unpolarized Q): for finite m_q, $\langle E_l/m_Q\rangle$ is softened by a factor $\simeq (1-m_{q'}^2/m_Q^2)$. In QCD perturbation theory, the q' produced by the W^* emission need not be on-shell, but may have a spectrum of invariant masses extending up to the kinematic boundary $\sim m_Q$, thus softening the l spectrum produced in the decay. Assuming $m_{q'} = 0$, the production of a lepton with energy fraction x requires that the total transverse momentum emitted in gluons by the outgoing q' be $k_T^2 \lesssim m_Q^2(1-x)$. This restriction forces any gluons emitted to be both soft and nearly collinear with the q', yielding double logarithmic terms. Keeping only leading terms at $x \to 1$, the $0(\alpha_s)$ correction becomes [19, 20]

$$\frac{d\Gamma}{dx} \simeq \frac{d\Gamma_0}{dx}\left(1-\frac{2\alpha_s}{3\pi}\log^2(1-x)\right), \tag{2}$$

(which implies a correction $\simeq (1-\alpha_s/\pi)$ to $\langle E_l/m_Q\rangle$) [F11]. Emissions of multiple soft and collinear gluons are independent, and their effects thus exponentiate when summed to all orders in α_s (in the leading double log approximation), yielding

$$\frac{d\Gamma}{dx} \simeq \frac{d\Gamma_0}{dx}\exp\left[-\frac{2\alpha_s}{3\pi}\log^2(1-x)\right] \tag{3}$$

(Since this approximation is formally valid only for $x \to 1$, modifications e.g., to $d\Gamma_0/dx$ by polynomials in x cannot be distinguished). Note that whereas the $0(\alpha_s)$ form (2) exhibited an unphysical divergence at $x = 1$, the exponentiated form (3) goes to zero as $x \to 1$

(after a peak at $x \simeq 1-\Lambda^2/m_Q^2$, corresponding to a produced q' invariant mass $\simeq \Lambda$). This radiation damping results from the impossibility of producing q' with no gluon emission. Eq. (3) assumes $m_{q'} = 0$: for $m_{q'} \gtrsim \Lambda$, divergences from collinear gluon radiation are regulated, and the correction factor becomes

$$\simeq \exp\left[\frac{4\alpha_s}{3\pi}\log(m_Q^2/m_{q'}^2)\log(1-x)\right] = (1-x)^{[4\alpha_s/3\pi\log^2(m_Q^2/m_{q'}^2)]}.$$

(An analogous result obtains in μ decay with $m_l \neq 0$ (e.g., Ref. [22]). The double log exponentiation (3) is known only to leading order: exponentiation of the single log terms appearing when $m_{q'} \neq 0$ is proven to all orders. For large m_Q, one may also estimate the total semileptonic decay rate from Fig. 3(a) using perturbation theory. To $0(\alpha_s)$, the correction due to gluon emissions is analogous to that in μ decay and given by $\Gamma = \Gamma_0\left(1-\left(\pi^2-\frac{25}{4}\right)\left(\frac{2\alpha_s}{3\pi}\right)\right) \simeq \Gamma_0(1-1.81(4\alpha_s/3\pi))$: integration of the approximate differential decay spectrum (2) yields $\Gamma \simeq \Gamma_0(1-1.84(4\alpha_s/3\pi))$ [F12]. The excellence of this approximation suggests an estimate of the correction summed to all orders in α_s by integration of (3), yielding [20] $\Gamma \simeq \sqrt{\pi/4A}\,\mathrm{erfc}(1/2\sqrt{A})\,\exp(-1/4A)\,\Gamma_0$, with $A = 2\alpha_s/3\pi$ and $\mathrm{erfc}(z) = 2/\sqrt{\pi}\int_z^\infty \exp(1-x^2)dx$ [F13]. Note that no infrared divergences appear at any order in α_s: they are cancelled by summation of all possible gluon configurations (real and virtual corrections) in the final state. (If the incoming Q were massless, then it could be degenerate with states containing in addition collinear gluons; uncancelled infrared divergences would remain unless the contributions of such states were included. The cancellation of infrared divergences associated with scattering or decay of a single incoming massive quark on summation only over possible final states has been proved to all orders in α_s [23]. The absence of infrared divergent terms sensitive to the structure of the initial state supports the approximation of an isolated initial Q. It is also noteworthy that the Γ/Γ_0 found here contains no uncancelled ultraviolet divergences or $0(\alpha_s\log(m_W/m_Q))$ terms: the W exchange is thus safely approximated by a four-fermion interaction. The reason for this is that gluon corrections to $Q \to q'l\nu$ act only at the $Q \to q'W^*$ vertex: they are ultraviolet finite by virtue of the QCD Ward identity (since W^* is color singlet) [F14], and cannot depend on m_W (but only on the W^* invariant mass $\sim m_Q$). The ultraviolet behavior of electromagnetic corrections to $A \to Bl\nu$ (e.g., $\mu \to \nu_\mu e\bar{\nu}_e$) is somewhat more complicated [24]: because l is charged, virtual photons may be exchanged between A and/or B and l, so that a γ^*W^* box diagram occurs. Such a diagram is ultraviolet divergent when $m_W \to \infty$ (local four-fermion interaction). However, in the particular case of μ decay with a $V-A$ coupling, the divergences and log (m_μ/m_W) terms cancel. This may be proved by showing that no divergences appear as $m_W \to \infty$. In this limit the local $(\mu\bar{\nu}_\mu)\,(\bar{e}\nu_e)$ interaction may freely be Fierz transformed and for $V-A$ W^* couplings, may be written as $(\mu\bar{e})\,(\bar{\nu}_\mu\nu_e)$, with the same $V-A$ couplings. This interaction may be pictured as occurring by exchange of an (infinite mass) fictitious neutral vector $W^{\circ *}$ between $\mu\bar{e}$ and $\bar{\nu}_\mu\nu_e$. Then photon corrections occur only at the $\mu\bar{e}W^{\circ *}$ vertex, and are ultraviolet finite because the V,A $W^{\circ *}$ current is (approximately [F14]) conserved (and clearly cannot depend on the fictitious m_{W°). If, however, the original coupling is not of the $V-A$ form, then Fierz rearrangement will introduce non-vector currents, which are not conserved, and usually lead to ultraviolet divergences (hence electromagnetic corrections to neutron decay with phenomenological n, p fields and a $\sim V-1.25$ A coupling are not ultraviolet finite). Further, only if in $A \to Bl\nu$ the electric charges $e_A = e_l$ is Fierz rearrangement to neutral currents possible. Electromagnetic corrections to $Q \to q'l\nu$ with e.g., $e_Q = 2/3$, $e_{q'} = -1/3$, $e_l = 1$ are not ultraviolet finite as $m_W \to \infty$: to obtain renormalizable corrections, one must use the complete Weinberg-Salam model in this case, and include, for example, photon emissions from the intermediate W^*. After renormalization, electromagnetic corrections to $Q \to q'l\nu$ may well contain $\sim \alpha \log(m_Q/m_W)$ terms.

The considerations of the previous paragraph were based on perturbation theory. Hadronic effects presumably become important when gluon emissions have degraded the final q' invariant mass to within $\sim \Lambda$ of its mass shell. The behavior of $d\Gamma/dx$ for $x \gtrsim 1-\Lambda/m_Q$ is particularly sensitive to this region, so that perturbative estimates become unreliable (and, for example, the peak at $x \simeq 1-\Lambda^2/m_Q^2$ from (3) with an effective coupling $\alpha_s(m_Q^2) \simeq 1.5/\log(m_Q^2/\Lambda^2)$ may be swamped by hadronic effects). Nevertheless, experimental electron spectra from D decays may be adequately fit by the perturbative estimate, but including a Fermi smearing ~ 0.3 GeV for the initial c (and taking $m_c \simeq 1.5$ GeV, $m_s = 0.5$ GeV [F15]): setting $\alpha_s = 0$ leads to a slightly worse fit. For example, the experimental [25] $\langle E_e/m_D\rangle \simeq 0.19 \pm 0.05$ while with $\alpha_s = 0$ one obtains $\langle E_e/m_D\rangle \simeq 0.35$ and with $\alpha_s \neq 0$ ($\Lambda = 0.5$ GeV) $\langle E_e/m_D\rangle \simeq 0.29$ to $0(\alpha_s)$ and $\langle E_e/m_D\rangle \simeq 0.32$ by estimating higher orders as in (3). For b-quark decays, hadronic effects should be comparatively unimportant: assumed a dominant bcW coupling, one may therefore make a firm prediction of $\langle E_e/m_B\rangle \simeq 0.28$ for this case (if $\alpha_s = 0$, it would become $\simeq 0.30$).

The discussion of semileptonic decays above has assumed the mechanism of Fig. 3(a). The process of Fig. 3(b) would yield different results. In direct analogy to the discussion of purely leptonic pseudoscalar meson decays above, the rate for Fig. 3(b) (with $m_l \ll m_Q$) vanishes by helicity suppression when the energy of the final hadrons goes to zero. To determine the recoil hadron energy spectrum, one must estimate what fraction of the initial M energy is effectively carried by the Q and $\bar{q}$ (when the M is "probed" by a W^* of invariant mass $\sim m_Q$). A rough guess would be that the gluons in M effectively carry an energy $\sim \Lambda$ [F19]. In that case, Fig. 3(b) would suffer a helicity suppression $\sim (\Lambda/m_M)^2$. In addition to such "primordial" gluons, the presence of the W^* interaction may induce gluon radiation whose rate may perhaps be estimated by perturbation theory. Starting from a color singlet pure $(Q\bar{q})$ system two gluons must be emitted to conserve color (so that $(Q\bar{q}) \to W^*GG$ occurs), although one of these gluons may be arbitrarily soft. In practice, the initial state must always contain some gluons albeit arbitrarily soft, which allow the annihilating $Q\bar{q}$ to be in a color 8 state, and thus require no second gluon emission. (Formally, the $(Q\bar{q}) \to W^*GG$ process exhibits an uncancelled infrared divergence [F16]; this is presumably cancelled when the appropriate composite initial state is included). I shall assume that the process may be estimated by considering only single hard gluon emission, and that the amplitude for the soft gluon processes which account for color conservation is one. Then the single gluon energy spectrum [26] $\sim 6x(1-x)$, and the total rate is given in analogy with $M \to l\nu\gamma$ by $\Gamma(M \to l\nu GX) \sim G_F^2 \alpha_s f_m^2 m_M^5/(720\pi^2 m_{\bar{q}}^2)$, where the effective $\bar{q}$ color magnetic moment is taken as $\sim g/m_q^{eff}$ [F17] ($\sim g/\Lambda$ for u, d). This rate is to be compared with the (free quark approximation) result $\Gamma(M \to l\bar{\nu}_l X) \sim \sim G_F^2 m_Q^5/(192\pi^3)$ for Fig. 3(a), after accounting for the different mixing angles (hence effective G_F) sampled in the two cases. For D decays, Fig. 3(b) is Cabibbo suppressed, while Fig. 3(a) is not. In F decays, on the other hand, neither Fig. 3(b) nor Fig. 3(a) suffers Cabibbo suppression. Taking the estimates $|\psi_F(0)|^2 \sim m_s^3$ given above suggests $\Gamma(F \to \to l\bar{\nu}_l GX) \sim G_F^2 \alpha_s m_M^5 (m_s/m_M)/(720\pi^2)$, which is not competitive with Fig. 3(a). An evident consequence of the mechanism of Fig. 3(a) is that semileptonic D^+, D° and F decays should be similar. (Experimental results support the similarity of lepton spectra [25] and probably total rates in semileptonic D^+ and D° decays). If Fig. 3(b) were important in F decay, then its semileptonic decay width would be larger than those of D^+ and D° (although if $F \to l\nu GX$ were important, the l energy spectrum would fortuitously probably be quite similar, with $\langle E_l/m_F \rangle \simeq 0.38$). For mesons containing b quarks, only $(b\bar{c})$ states may undergo semileptonic decays through Fig. 3(b) without additional mixing angle suppression: but there the ratio of Fig. 3(b) to Fig. 3(a) is probably still smaller than for D, F decays.

One may be tempted to apply the basically perturbative analysis of semileptonic decays given above to K decays. The K mass is, however, far too small for such a procedure to be profitable: one should instead consider explicit exclusive hadronic modes, such as $K \to \pi l\bar{\nu}_l$. In describing exclusive semileptonic decays, one must sum coherently [F18] over possible quark subprocesses: by construction, all subprocesses yield the same final state. Decays of the form $M' \to Ml\nu$ (which for zero W^* invariant mass occur only through the vector weak current) may often be related to each other and to electromagnetic processes using approximate SU(3) invariance. For example, isospin invariance constrains $\Gamma(K^+ \to \to \pi^\circ e^+ \nu) = 2\Gamma(K_L^0 \to \pi^+ e^- \nu)$ (up to $0(1\%)$ phase space and electromagnetic radiative corrections). However, for decays such as $K \to \pi\pi e\nu$ the number of possible contributing amplitudes precludes such relations. The mechanism of Fig. 3(a) suggests that $\Gamma(K^+ \to \to \pi\pi e\nu) = \Gamma(K^\circ \to \pi\pi e\nu)$: on the other hand, the process of Fig. 3(b) can occur (with "valence" quarks) only for K^+ (yielding an $I = 0$ $\pi\pi$ state), suggesting that the rates for the K^+ and K° decay may differ. Fig. 3(b) probably suffers little helicity suppression with respect to Fig. 3(a) in this case. (Experimentally, $K^+ \to \pi\pi e\nu$ has a measured branching ratio $\simeq 6\times10^{-5}$, while the $K_L^0 \to \pi\pi e\nu$ branching ratio is $\lesssim 2\times10^{-3}$; it would be expected at a level $\simeq 3\times10^{-4}$).

In Fig. 3(a), Q is not necessarily the heaviest quark in M. For example, a $(b\bar{c})$ meson may decay either through $b \to cW^*$ (to $c\bar{c}W^*$) or through $\bar{c} \to \bar{s}W^*$ (to $b\bar{s}W^*$), where in the latter case, a decay $b \to cW^*$ follows. Unless mixing angles strongly inhibit decays of the heavier quark, these will, however, probably dominate (for the $(b\bar{c})$ case, $\Gamma(b \to \to cW^*)/\Gamma(\bar{c} \to \bar{s}W^*) \sim (m_b/m_c)^2 \sin^2\theta_{bc} \gg 1$).

The discussion of semileptonic decays above has implicitly assumed that the decaying meson mass $m_M \ll m_W$. When m_M approaches m_W, the semileptonic M decay rate (from Fig. 3(a)) will increase, as the intermediate W^* becomes closer to its mass shell: in the limit $m_Q \to m_W$, Γ is enhanced by a factor six through this effect. When $m_M \gtrsim m_W$, decays to real W should dominate (since they involve fewer powers of the semiweak coupling constant): the resulting three hadron jets should provide a spectacular signature for production of such heavy M, e.g., in $\bar{p}p$ collisions [28]. A simple calculation of the rate for the decay $Q \to q'W$ indicates that it grows $\sim \alpha m_Q^3/m_W^2$, due to the production of longitudinally-polarized W, so that for $m_Q \gtrsim 150$ GeV, a produced Q would undergo weak decay before being confined into a pseudoscalar meson: it could therefore, for example, transmit polarization information from its production to its decay products. However, the presence of fermions with $m \gtrsim 150$ GeV is not consistent [29] with the standard weak interaction model used to calculate the Q decay rate. The semileptonic decays considered above were all of pseudoscalar mesons. However, for $m_Q \gtrsim 30$ GeV, vector ζ (i.e., $(Q\bar{Q})$) states should decay through $Q\bar{Q} \to q'\bar{Q}W^*_{\to l\nu}$ with an $0(10^{-4}$–$10^{-2})$ branching ratio [30]. As mentioned above, ζ with masses close to m_Z should exhibit large $\zeta \to Z^* \to l\bar{l}$ branching ratios.

Non-Leptonic Decays

Fig. 4 shows schematically various processes which contribute to weak nonleptonic decays of mesons M [F20]. The relative importance of these mechanisms varies widely between different decays, in principle allowing their effects to be disentangled. In addition, for sufficiently large m_M, it should be possible to distinguish directly between the processes of Fig. 4 by the structure of the final states that they yield [31] (for example, Fig. 4(a) would lead to predominantly three-jet final states, while Fig. 4(c) would usually give

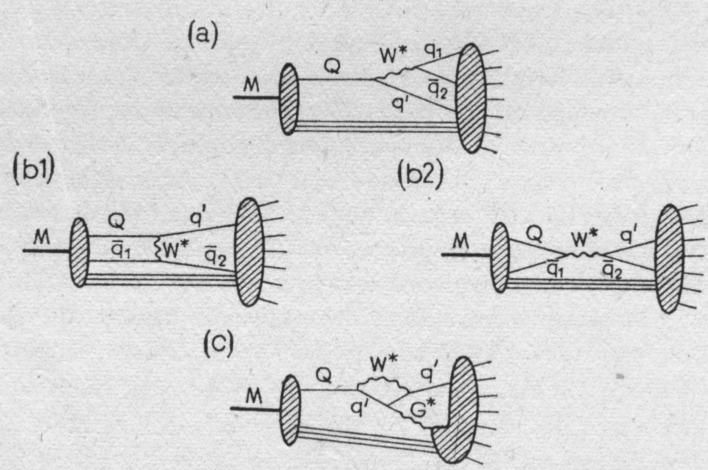

Fig. 4. Schematic diagrams for nonleptonic decay of a $(Q\bar{q})$ meson M.

rise to two-jets). It will turn out that all the mechanisms in Fig. 4 should contribute to K decays. Non-leptonic D^0 decays are probably dominated by Fig. 4(b); in F^+ decays, Figs. 4(a) and 4(b) are probably competitive, and in D^+ decays Fig. 4(a) probably dominates. For heavier mesons, Figs. 4(b, c) should progressively become less important.

In the free quark approximation, Fig. 4(a) implies an M decay width $\Gamma_a^0 = \Gamma(Q \to q'q\bar{q}) \simeq G_F^2 m_Q^5 (3N_q)/(192\pi^3)$, where $(3N_q)$ gives the effective number of $q\bar{q}$ generations to which the intermediate W^* may decay and $0(m_{q'}^2/m_Q^2)$ corrections from non-zero final quark masses have been ignored. Any mixing angle factors appearing at the $Q \to q'W^*$ vertex have been absorbed into an effective G_F. Note that existing examples (c, b) suggest that these mixing angles arrange all but $\lesssim 20\%$ of all Q decays to be to the q' closest in mass to Q (although it appears that $m_{q'}/m_Q \lesssim 1/3$). Combining Γ_a^0 with the semileptonic decay rate implied by the analogous diagram Fig. 3(a) (which should always dominate over Fig. 3(b), except perhaps for some K decays, as discussed above) yields in the free quark approximation a semileptonic branching ratio $\Gamma(M \to lX)/\Gamma(M \to X)$ given just by the W^* semileptonic branching ratio $\Gamma(W^* \to lX)/(M \to X) \simeq N_l/(3N_q+N_l)$. Thus Fig. 4(a) implies that the semileptonic branching ratios for all mesons containing a given heavy quark Q should be equal. The large experimental violation of this equality in s and c decays demonstrates that these receive important contributions from processes other than Fig. 4(a) (see below); for heavier quarks, however, Fig. 4(a) should dominate. If mixing angles always arrange q' to be closest in mass to Q, then decay of a very heavy Q through Fig. 4(a) would involve many W^* emissions: the probability that a lepton would be produced in one of these W^* decays is quite high ($\sim 1-[3N_q/(3N_q+N_l)]^N$ for an Nth generation Q). Gluon emissions in QCD modify the total width for Fig. 4(a) obtained in the free quark approximation. Just as for Fig. 3(a), the corrections are infrared finite by virtue of the non-zero initial Q mass. To $0(\alpha_s)$ color averaging decouples the contributions from gluons emitted by the W^* decay products and by the q' (interference diagrams are proportional to the color matrix $T_{ii}^a \equiv 0$, since W^* carries no color). Hence, to $0(\alpha_s)$, $\Gamma(M \to lX)/\Gamma(M \to X)$ remains proportional to the W^* semileptonic branching ratio $\simeq N_l/(3N_q(1+\alpha_s/\pi)+N_l)$. This corresponds to an $0(\alpha_s)$ correction $\Gamma/\Gamma_0 \simeq 1-(\alpha_s/3\pi)\times$ $\times(2\pi^2-31/2) \simeq 1-0.45\alpha_s$ (Ref. [31], p. 447; Ref. [32]), to the Q non-leptonic decay rate. Note that to this order, the correction is finite as $m_W \to \infty$: in higher orders, $\log(m_W/m_Q)$ terms may appear (although all ultraviolet divergences must be renormalizable within QCD alone, since W^* is colorless). The decoupling of contributions from gluon emissions from q' and q_1, $\bar{q}_2$ does not persist beyond $0(\alpha_s)$: the $0(\alpha_s^2)$ correction must be obtained by an explicit (rather complicated) calculation [F21]. The result will determine the relevant scale for the effective coupling α_s appearing in Γ/Γ_0 to $0(\alpha_s)$: this scale is presumably $0(m_Q)$ rather than $0(m_W)$, but may involve large numerical constants. In going beyond $0(\alpha_s^0)$ one must presumably account for gluon exchanges between the Q (or its decay products) and the "spectator" $\bar{q}$ in the original M. To $0(\alpha_s)$, color averaging again cancels diagrams involving a gluon exchanged between the W^* decay products and the $\bar{q}$: hence to $0(\alpha_s)$, spectator effects do not affect the semileptonic branching ratio. The rôle of the "spectator" $\bar{q}$ is similar to that of atomic electrons in nuclear beta decay: so long as the momenta of the Q decay products are much greater than the inverse size ($\sim \Lambda$) of the initial M state, the spectator's effects will probably be negligible [F22]. If one of the Q decay products is correlated (in flavor, color and momentum) with the spectator, then interference (e.g., Pauli exclusion) effects may occur [F23]. Since the average momentum of the Q decay products $\sim m_Q/3$, they should lie in a different region of phase space from the spectator, with negligible interference, unless $m_Q \lesssim 2$ GeV. When $c \to s u \bar{d}$ occurs in a $D^+ \equiv c\bar{d}$ meson, the two final antiquarks have the same flavor; in a $D^\circ \equiv c\bar{u}$ meson, they do not. The nature of the interference between the two $\bar{d}$ in D^+ decay depends on their colors: if the $SU(3)_{flavor}$ properties of the decay are unrestricted, then the Pauli exclusion between $\bar{d}$ with identical colors enhances the total D^+ decay rate (cf., Ref. [33]). The same result obtains if one treats the final state as two color singlet $q\bar{q}$ systems, and allows interference between production of the color singlet systems, even if the colors of their constituent $q, \bar{q}$ differ (i.e., $A\bar{A}$ and $B\bar{B}$ systems are treated as indistinguishable, although their internal quarks have different colors). If now, the $SU(3)_{flavor}$ properties of the final state are restricted, other patterns of interference are obtained: for example, if the $c \to \to su\bar{d}$ effective Hamiltonian transforms as *6* under $SU(3)_{flavor}$ (so that it is antisymmetric under the interchange $s \leftrightarrow u$), then destructive interference occurs [34] if the quark momenta are similar. Similarly, in K^+ decays, the $|\Delta I| = 1/2$ components may be suppressed by such interference effects. Up to $O(\alpha_s)$ corrections the three final quarks in Fig. 4(a) should give rise at sufficiently high m_W to three separated hadron jets: the spectator $\bar{q}$ carries only a small fraction ($\sim\Lambda/m_Q$) of the total M energy for large m_M, and so cannot initiate a jet. In the free quark approximation, production of an $M\bar{M}$ pair [F30] at rest (threshold) with decays according to Fig. 4(a) gives the shape parameters [31] $\langle H_2\rangle = (119/2 - -6\pi^2) \simeq 0.28$, $\langle H_3\rangle = 0.12$. For comparison, a two-particle final state gives $\langle H_2\rangle = = 1$, $\langle H_3\rangle = 0$, while an isotropic final state has $\langle H_2\rangle = \langle H_3\rangle = 0$. (A three-body final state with uniform matrix element over available phase space gives $\langle H_2\rangle = (3\pi^2 - 29) \simeq \simeq 0.61$, $\langle H_3\rangle = 0.21$). Two-jet decays, as may result from Fig. 4(c), would give $\langle H_2\rangle = = 1/2$, $\langle H_3\rangle = 0$. Subsequent decays of heavy quarks produced in these decays should usually not modify the $\langle H_l\rangle$ significantly (since $m_{q'}/E_{q'}$ is typically $\lesssim 1$, the secondary decay products do not subtend large angles), although identification of the decays would reveal the flavor of the quark jet, and thus aid in discriminating between the processes of Fig. 4. The $\langle H_l\rangle$ in the free quark approximation quoted above are modified by gluon emissions from the final partons, and by hadronic effects. Simulations [31] of these corrections suggest that for $m_Q \gtrsim 3$ GeV, $e^+e^- \to M\bar{M}$ processes near threshold should be clearly distinguished from the usual two-jet $e^+e^- \to q\bar{q}(GG\,...)$ processes by the comparative isotropy of their final states, as revealed for example in the H_l distributions $1/\sigma\cdot \cdot d\sigma/dH_l$. For $m_Q \gtrsim 18$ GeV, it appears that the three-jet final states of Fig. 4(a) should also be distinguished from the two-jet ones of Fig. 4(c), allowing direct discrimination between these decay mechanisms for mesons containing t quarks.

Consider now the process illustrated in Fig. 4(b), which is analogous to (though much more effective than) the semileptonic decay mechanism of Fig. 2(b). In contrast to Fig. 4(a), the contributions of Fig. 4(b1) and Fig. 4(b2) depend critically on the quantum numbers of the initial M state. At momenta $\ll m_W$, the Lorentz structures of the diagrams (b1) and (b2) are identical (they are related in this limit by Fierz reordering of the effective four-fermion vertices). The diagrams differ in flavor, and also in that Fig. 4(b2) requires the annihilating $Q, \bar{q}$ to be in a color singlet state, while Fig. 4(b1) does not. As discussed above for Fig. 2(b), the inevitable presence of soft gluons from the initial M state means that the initial $Q, \bar{q}$ populate all color states according to their statistical weights. (Summing over final colors in Fig. 4(b2) gives a factor 3; the necessary averaging over the initial colors introduces a cancelling factor 1/3, so that Figs. 4(b1) and Fig. 4(b2) do not differ in their color combinatoric weights [F24]). For s-quark decays, Fig. 4(b1) contributes in $s\bar{d}$ (K°) mesons and Fig. 4(b2) in $s\bar{u}$ (K^-) mesons [F25]. For c-quark decays, Fig. 4(b1) can contribute in $c\bar{u}$ (D°) mesons, but Fig. 4(b2) is Cabibbo suppressed by a a factor $\tan^2\theta_c \sim 0.04$ in $c\bar{d}$ (D^+) mesons; it can nevertheless contribute to $c\bar{s}$ (F^+) meson decays. For b-quarks, a similar pattern obtains: $(b\bar{d})$ and $(b\bar{s})$ mesons may decay through Fig. 4(b2) without additional mixing angle suppression relative to $b \to cW^*$ in Fig. 4(a); only $(b\bar{c})$ and not $(b\bar{u})$ mesons may decay without suppression through Fig. 4(b1). This pattern would continue for t quarks. In the free quark approximation, the rate for decay of a pseudoscalar meson by the processes Fig. 4(b1) and Fig. 4(b2) may be estimated in analogy to Fig. 1 as $\Gamma^0_{b1} \simeq G_F^2 f_M^2 m_M^3 (m_{q'}/m_M)^2/(4\pi)$ and $\Gamma^0_{b2} \simeq G_F^2 f_M^2 m_M^3 (m_{q'}/m_M)^2 \cdot$ $\cdot (N_q)/(4\pi)$, where mixing angle factors have again been absorbed into G_F. The factor $(m_{q'}/m_M)^2$ arises through helicity suppression, as in Fig. 1. These decay rates are to be compared with the rate $\Gamma^0_a \simeq G_F^2 m_Q^5\ (3N_q)/(192\pi^3)$ for the three-body Q decay Fig. 4(a). The ratio of Γ^0_b to this is $\{\Gamma^0_{b1}, \Gamma^0_{b2}\}/\Gamma^0_a \simeq 200\ (m_q/m_Q)^2 (f_M/m_Q)^2 \{(1/N_q), 1\}$: the factor 200 results from suppression of Fig. 4(a) relative to Fig. 4(b) by virtue of its three-body final state. As discussed above, for mesons containing a light (u, d, s) spectator $\bar{q}$, f_M probably $\sim \Lambda$. Thus, even if m_q/m_W always $\sim 1/3$, Γ^0_b/Γ^0_a would fall below ~ 1 for $m_Q/\Lambda \gtrsim 3$: however, for comparatively small m_Q, Fig. 4(b) may well dominate over Fig. 4(a). In K decays, Fig. 4(b) should be at least comparable in importance to Fig. 4(a) (the naive free quark estimate suggests that it should dominate). As mentioned above, the contribution of Fig. 4(b2) is Cabibbo suppressed in D^+ decay (so that it is unimportant in the total D^+ width, but should be significant in the partial width for $S = 0$ final states, as discussed below). In D° decays, $m_{q'} \simeq m_s$, and $\Gamma^0_{b1}/\Gamma^0_a \sim 200\ (m_s/m_c)^2\ (f_D/m_D)^2$. Even with pessimistic choices for m_s and f_D, this ratio is $0(1)$, and it is very possibly $\gg 1$. In this case, the D° decay width would be dominated by Fig. 4(b1), and be larger than the D^+ width, which is dominated by Fig. 4(a). The experimental observation (discussed below) $\Gamma(D^\circ)/\Gamma(D^+) \gg 1$ supports this picture. For F^+ decays, $m_{q'} \simeq m_u^{eff}$, so that Γ^0_{b2} there is slightly smaller than Γ^0_{b1} in D° decays. In $(b\bar{d})$ decays, $m_{q'} \simeq m_c$: if $f_M \sim \Lambda$ in this case, then $\Gamma^0_b/\Gamma^0_a \ll 1$. In $(b\bar{c})$ decays, where presumably $|\psi(0)|^2 \sim m_c^3$, Γ^0_b/Γ^0_a is again $0(1)$. When $\Gamma^0_b > \Gamma^0_a$, final states of M decays will consist predominantly of two jets $(q'\bar{q})$. QCD effects will modify the free quark estimate Γ^0_b. To $0(\alpha_s)$, however, no $0(\alpha_s \log(m_W/m_Q))$ terms appear, so long as the colors of the initial and final quarks are summed over without restriction. The reason is as for Fig. 3(a): color averaging cancels diagrams in which the W^* exchange is accompanied by a gluon; for the remaining diagrams the W^* exchange may be approximated by a local four-fermion vertex, since only the W^* invariant mass (and not m_W itself) is relevant. If the colors of initial or final quarks are restricted, then $0(\alpha_s \log(m_W/m_Q))$ terms do appear: these will be crucial in consideration of the $|\Delta I| =$ $= 1/2$ rule below. In higher orders, $0(\alpha_s^2 \log(m_W/m_Q))$ corrections may appear even for color-averaged widths. Exchanges between the incoming Q, $\bar{q}$ lead to infrared divergences: at least in as far as $0(\log(m_W/m_Q))$ terms are absent, these are, however, subsumed in the definition of f_M or $|\psi(0)^2$ [F26]. Gluon emissions and exchanges in the final q, $\bar{q}$ state modify the total width through their effect on the W^* decay width. Another potentially important effect of QCD corrections is in modifying or circumventing the helicity suppression factor in Γ_b^0. The possibility for the final quarks to be produced with large invariant masses before radiating gluons might be expected to affect the helicity suppression. In fact, these effects lead only to $0(\alpha_s)$ corrections: the divergence of the weak current, to which a spin-0 W^* must couple, remains zero to all orders in α_s if the quark masses vanish, irrespective of the W^* invariant mass. (Nevertheless, confinement effects can contribute terms $0(\Lambda^2/m_Q^2)$ from the $0(\Lambda)$ effective quark masses they produce). If the helicity suppression is genuinely to be avoided, then, as discussed above for $M \to l\nu\gamma$ and Fig. 3(b), gluon emissions must produce a spin-1, rather than spin-0, $Q\bar{q}$ state. As for Fig. 2(b), however, such effects are probably not large [F27]. In analogy with Fig. 3(b), one may perhaps estimate by perturbation theory the rate for a gluon emission to produce a spin-1 state as $\Gamma_b^1 \sim G_F^2\alpha_s m_M^3 f_M^2/(720\pi^2)\,(m_M/m_{\underline{q}}^{eff})^2$ [F28]. Here, the initial light quark mass enters through its magnetic moment, and thus appears in the denominator: in Γ_b^0, however, the final q' mass appeared in the numerator, providing helicity suppression. Thus $\Gamma_b^1/\Gamma_b^0 \sim$ $\sim (\alpha_s/180\pi)m_M^4/(m_{q'}^2, m_{\underline{q}}^2)$: taking $m_{\underline{q}}^{eff} \sim \Lambda \sim 0.5$ GeV (as appropriate for u, d, s quarks), then if $m_{q'} \sim 0.5$ GeV, $m_M \gtrsim 4$ GeV (or perhaps $m_M \lesssim 1$ GeV) is necessary before $\Gamma_b^1/\Gamma_b^0 \gtrsim 1$. (If $m_{q'} \sim m_M/3$, then $m_M \gtrsim 9$ GeV is required). Estimating $|\psi(0)|^2 \sim [m_{\underline{q}}^{eff}]^3$ gives $\Gamma_1^0 \sim (10^{-4}\alpha_s)G_F^2 m_M^2 m_{\underline{q}}^{eff}$, so that $\Gamma_b^1/\Gamma_a^0 \sim \alpha_s(m_{\underline{q}}^{eff}/m_M) \ll 1$. Thus, while gluon emission can avoid helicity suppression, it cannot render Fig. 4(b) competitive with Fig. 4(a) at large $m_Q/m_{q'}$.

I now discuss the diagrams of Fig. 4(c) (sometimes referred to by the inappropriate name of "penguins" [35]). In these diagrams, the final q'' has the same electric charge as Q, and therefore must lie in a different weak isomultiplet: the dominant contributions come when the intermediate q' is in either the Q or the q'' isomultiplet. For s-quark decays, the intermediate q' is predominantly u or c: in either case Fig. 4(c) is proportional to the mixing angle factor $\sin\theta_c\cos\theta_c$. However, for s-quark decays, Fig. 4(a) is also proportional to $\sin\theta_c$, so that in this case, Fig. 4(c) may be competitive with Fig. 4(a). For c-quark decays, the intermediate q' in Fig. 4(c) is s or d, and the final q'' is u, so that again a mixing angle factor $\sim \sin\theta_c \cos\theta_c$ enters. On the other hand, in c-quark decays, Fig. 4(a) gives an $|S| = 1$ final state, with mixing angle factor $\sim \cos\theta_c$: in this case, therefore, Fig. 4(c) may only be significant in suppressed $S = 0$ final state decays. For b-quark decays, intermediate t and c in Fig. 4(c) yield $b \to sG^*$ transitions; these involve essentially the same mixing angle factors as $b \to cW^*$ in Fig. 4(a), but lead to $|S| = 1$, $C = 0$ rather than $|C| = 1$, $S = 0$ final states. t-quark decays are similar to c-quark ones: Fig. 4(c) suffers mixing angle suppression with respect to Fig. 4(a), and may only be important in suppressed decay channels. This pattern would probably be repeated for any further generations of quarks: the heavier member of each weak isodoublet would decay predominantly to the lighter member, with little contribution from Fig. 4(c), while in decays of the lighter member, Figs. 4(a) and 4(c) may be competitive.

In s-quark decays, the contributions of intermediate u and c contain mixing angle factors with opposing signs ($\cos\theta_c \sin\theta_c$ and $-\cos\theta_c \sin\theta_c$, respectively) by virtue of the GIM arrangement. If $m_u = m_c$, these contributions would cancel exactly, and Fig. 4(c) would vanish. Typically, if a momentum $|k| \gg m_c$ flows through the virtual intermediate q' line in Fig. 4(c), such a cancellation is effective: the magnitude of Fig. 4(c) is thus dominated by the region where the intermediate q' has a small momentum $|k| \lesssim m_c$, which is particularly sensitive to hadronic effects. For c decays, the d and s contributions typically cancel for $|k| \gtrsim m_s$, while in b decays, the t and c contributions only cancel for $|k| \gtrsim m_t$. These GIM cancellations also relegate the real gluon emission process $Q \to q''G$ to subleading order, and allow only virtual gluon emission. Consider the decay $s \to dG$, which is in many respects analogous to $s \to d\gamma$. If G is on its mass shell, this must be a magnetic moment transition $\sim M\bar{s}\sigma_{\mu\nu}q^{\nu}G^{\mu}d$ [F29] (q is the G momentum), even if s, d are off-shell: it is therefore proportional to the "s-d weak transition color magnetic moment". This is analogous to the weak contribution to the μ anomalous magnetic moment $\sim G_F m_\mu/\pi^2$ [F31]. However, in $s \to dG$, an intermediate u gives $\sim g_s G_F(m_s - m_d)\cos\theta_c \sin\theta_c$, while an intermediate c gives $\sim g_s G_F(m_s - m_d)(-\cos\theta_c \sin\theta_c)$, which cancels the u contribution up to terms $0(\alpha_s, m_c^2/m_W^2)$. The one-loop vertex diagram for $s \to dG$ (with momentum p for s) involves the numerator factor $\gamma_\lambda P_L(p+m)\gamma_\nu((p-q)+m)\gamma_\lambda P_L$, where $P_L \equiv (1-$ $-\gamma_5)/2$ is the left-hand projection operator from the W couplings, and m is the intermediate u, c mass: this factor contains no terms linear in m. Thus the mass parameter M appearing in the decay amplitude $\sim M\bar{s}\sigma_{\mu\nu}q^{\nu}G^{\mu}d$ cannot depend on the intermediate u, c masses (except through $0(m_{u,c}^2/m_W^2)$ corrections), but only on the external s, d masses, so that the u, c contributions differ (to leading order) only by mixing angle factors, and cancel as described above. This cancellation may be avoided, however, if one of the two possible intermediate q' has a mass $m_{q'} \gg m_W$, so that its effects are $0(m_W/m_{q'})$: in that case, the other possible q' is effectively freed from the GIM arrangement, and may mediate a decay $Q \to q''G$ at a substantial rate $\Gamma(Q \to q''G) \sim (8\alpha_s/3\pi)\Gamma(Q \to q'q\bar{q})$, (omitting mixing angle factors). If $m_t \gg m_M$ this interesting possibility would be realized in b-quark decays [F32]. The cancellation would also be avoided if W coupled not only to left-handed, but also to right-handed currents. In that case, one of the P_L in the numerator factor is replaced by $P_R(\equiv (1+\gamma_5)/2)$, and a term linear in the intermediate quark mass appears [36, 37, 38] (if such right-handed currents contributed in $s \to dG$ they would give a large amplitude $\sim g_s(m_c - m_u)G_F\bar{s}\sigma_{\mu\nu}q^{\nu}G^{\mu}d$). Any quarks coupled to such right-handed currents should decay at a rate $\Gamma(Q \to q''G) \sim (8\alpha_s/3\pi)(M/m_Q)^2\Gamma(Q \to q'q\bar{q})$ (dropping mixing angle factors), where M ($\lesssim m_W$) is roughly mass of the heaviest contributing intermediate q'. Charged Higgs scalars in place of the usual W may also lead to significant $Q \to$ $\to q''G$ decays [39]. In known cases, however, such processes are absent and $Q \to q''G$ decays are relegated to subleading order. The cancellation between u and c intermediate states in, for example, $s \to dG$, is not exact, but is violated by $0(m_c^2/m_W^2)$, yielding a negligible decay amplitude [38] $\sim (m_d+m_s)[(m_c^2-m_u^2)/m_W^2]G_F\sin\theta_c\cos\theta_c g_s/(12\pi^2)[2\log(m_W^2/m_c^2)-$ $-1/3]s\sigma_{\mu\nu}q^{\nu}G^{\mu}d$, assuming $m_c \lesssim m_W$. The cancellation between u and c intermediate states in $s \to dG$ at $0(g_s)$ relies on the absence of $0(\log(m_W/m_{q'}))$ terms from divergences in the loop integration. At $0(g_s^3)$, gluon corrections to $s \to dG$ introduce ultraviolet divergences and $0(\log(m_W/m_{q'}))$ terms (similarly, the weak contribution to $g_\mu - 2$ presumably receives electromagnetic corrections $\sim G_F m_\mu^2 \alpha \log(m_W^2/m_\mu^2)$): these destroy the cancellation and lead to a $s \to dG$ decay amplitude [40] $\sim (m_s - m_d)G_F\sin\theta_c\cos\theta_c(\alpha_s/4\pi)(g_s/16\pi^2)\times$ $\times(3/8)\log(m_c^2/m_u^2)\bar{s}\sigma_{\mu\nu}q^{\nu}G^{\mu}d$. Again this is in most cases probably too small to be of practical relevance. Just as independent higher-order corrections to $s \to dG$ can avoid the cancellation, so also can exchanges between the intermediate q' and spectator $\bar{q}$, by providing and additional spin-flip amplitude in the effective q' propagator. Soft vector exchanges are helicity-conserving and thus inadequate. However, the q' presumably propagates in an effective external color field generated by the $\bar{q}$ and other constituents of the original M. This may contain a spin-flip $\sim \vec{\mu}_{q'} \cdot \vec{B} \sim g_s[m_q^{eff}]^2/m_{q'}$ component, which may act over the propagation time of the virtual q'. For $s \to dG$, this may give a decay amplitude perhaps $\sim [m_q^{eff}]^2/(m_u^{eff}\Lambda)g_s^2\bar{s}\sigma_{\mu\nu}q^\nu G^\mu d$, which could be significant [F33]. Insofar as $Q \to q''G$ is small, so also the weak radiate decay process $Q \to q''\gamma$ should be small, as discussed below.

The GIM cancellation in Fig. 4(c) does not occur for $Q \to q''G^*$ if the radiated gluon is off its mass shell: the "s-d weak transition charge radius" is non-zero at leading order. Since the process $Q \to q''G^*$ via a one-loop W vertex correction exhibits no additional ultraviolet divergences in the limit $m_W \to \infty$, one may approximate the W exchange by a local four-fermion interaction. Then, performing a Fierz transformation, $Q \to q''G^*$ may be approximated by $Q \to q''$ "Z^{o*}", followed by "Z^{0*}" $\to G^*$ via a virtual q' loop. Thus the rate for $Q \to q''G^*$ is proportional to the one-loop q' vacuum polarization diagram [41]: taking k as the G^* momentum, the amplitude for, say, $s \to dG^*$ becomes $\sim G_F(\sin\theta_c\cos\theta_c)k^2F(k^2)\bar{s}\gamma_\mu G^\mu d$, with the form factor $F(k^2)$ given by the difference between one-loop vacuum polarization with a c and a u loop. For $k^2 \ll m_c^2, m_u^2, F(k^2) \simeq \simeq (g_s/16\pi^2)(1/6)\,[\log(m_s^2/\mu_R^2)-\log(m_u^2/\mu_R^2)]+0(k^2/m^2) \sim \log(m_c/m_u)$ [F34]; for $m_u^2 \ll \ll k^2 \ll m_c^2, F(k^2) \simeq (g_s/16\pi^2)(k^2/30m_u^2)$; and for $k^2 \gg m_c^2, m_u^2 F(k^2) \simeq (g_s/16\pi^2)(m_c^2 - - m_u^2)/k^2$ [F35]. The virtual gluon produced in $Q \to q''G^*$ may either be spacelike, and be absorbed by a spectator $\bar{q}$ (or q) from the initial meson, or be timelike, and "decay" into a $q\bar{q}$ pair. The presence of an explicit k^2 factor in the $Q \to q''G^*$ amplitude for small k^2 (which is necessary to maintain gauge invariance, since there can be no static $Q \to q''$ "transition charge") cancels the $1/k^2$ pole in the virtual G^* propagator, leading to an effective local $Q\bar{q} \to q''\bar{q}$ or $Q \to q''q\bar{q}$ interaction. An important difference from Figs. 4(b) and 4(a) is that the G^* coupling is independent of the q, $\bar{q}$ helicities, so that the effective local four fermion interaction contains terms with $(V-A)\times(V+A)$ as well as $(V-A)\times(V-A)$ Lorentz structure [F36]. Using the $s \to dG^*$ vertex given above, one may estimate the rate for the decay $s \to dq\bar{q}$ (integrating over the intermediate G^* invariant mass) as $\Gamma(s \to dq\bar{q}) \sim G_F^2 m_s^5/(192\pi^3)\,(1/36)\,(\alpha_s/4\pi)^2(m_s^2/30m_u^2)$ contains $(\sin\theta_c\cos\theta_c)^2$ (assuming $m_s \gg m_u \gtrsim \gtrsim m_q$), which is much smaller than the analogous rate from Fig. 4(a). For b quark decays, m_b^2/m_c^2 is again not sufficiently large to render this decay mechanism important. On the other hand, the process $Q\bar{q} \to q''\bar{q}$ may well be important for small m_Q. The intermediate G^* coupling is independent of the $\bar{q}$ helicity. The Q and q'' must be left-handed in order to participate in the weak vertex: however, the $\bar{q}$ may be right-handed, so that the initial and final $q\bar{q}$ systems have helicity zero. The spin of the decaying pseudoscalar M usually constrains the $q\bar{q}$ to have spin 0. In Figs. 2 and 4(b) the W^* coupling required both q and $\bar{q}$ to be left-handed giving a helicity 1 $q\bar{q}$ state with a helicity suppressed amplitude $0(m_q/E_q)$ to have spin 0. This helicity suppression is absent in Fig. 4(c) both for the initial and final $q\bar{q}$ systems. Not only is the explicit $0(m_f^2/E_f^2)$ factor in the rates for Figs. 2 and 4(b) absent for Fig. 4(c); in addition, the effective f_M^0 for Fig. 4(c) should be larger than f_M for Figs. 2

and 4(b) since it involves no helicity suppression factor: naively $f_M^0 \sim m_M/(m_q+m_{\bar{q}})f_M \sim$ $\sim [|\psi_M(0)|^2/m_M]$ [F37], [42]. For heavy M, there should be no significant enhancement of f_M^0 over f_M. For light M, the magnitude of the relevant m_q is difficult to estimate [43], but f_K^0 may be enhanced by perhaps even a factor ~ 10 over f_K (a naive guess would take $|\psi_K(0)|^2 \sim 1/r_K^3 \sim m_{K^*}^3$, suggesting $f_M^0/f_M \sim m_{K^*}^{3/2}/(m_K^{1/2}f_K) \sim 9$), which would render Fig. 4(c) dominant over Figs. 4(a, b) in K decays. To estimate the rate for $Q\bar{q} \to q''\bar{q}$, one must determine the invariant mass k^2 of the t-channel exchanged G^*. Assuming the initial $Q, \bar{q}$ to be at rest, the momenta of the final q'', $\bar{q}$ are completely determined, and $k^2 = -(m_Q-m_{q''})(m_Q+m_{q''})m_{q'}/(m_Q+m_{q'}) \simeq -m_Q m_{q'}$ [F38] (cf. [41]). The magnitude of the $Qq'G^*$ vertex form factor depends on the relative size of $k^2 \simeq -m_Q m_{q'}$ and the intermediate virtual q' mass. In s quark decays, $m_{q'} = m_u$, m_c while $k^2 \simeq -m_s m_{u,d}$. Hence $F(k^2) \simeq (g_s^2/16\pi^2)(m_s/30m_u)$, yielding a naive free quark estimate $\Gamma(s\bar{q} \to d\bar{q}) \simeq$ $\simeq (\alpha_s/120\pi)^2 G_F^2(m_s/m_u)^2(\sin\theta_c\cos\theta_c)^2[f_K^0]^2 m_K^3/8\pi$: the small numerical factors appearing in $F(k^2)$ render this estimate, like that for $\Gamma(s \to dq\bar{q})$ above, slightly below the estimated $\Gamma(s \to dq\bar{q})$ from Fig. 4(a). In c quark decays, $c\bar{q} \to u\bar{q}$ contributes only to Cabibbo suppressed $S = 0$ final state decays. Here $m_{q'} = m_d, m_s$ and $k^2 \simeq -m_c m_u$ (for D decays) $\simeq$ $\simeq -m_c m_s$ (for F decays) [41] so that $F(k^2) \simeq (g_s/16\pi^2)(m_s^2-m_d^2)/(m_c m_{\bar{q}})$, yielding a naive estimate $\Gamma(c\bar{q} \to u\bar{q}) \simeq (\alpha_s/4\pi)^2 G_F^2(m_s^2/(m_c m_{\bar{q}}))^2(\sin\theta_c\cos\theta_c)^2[f_D^0]^2 m_D^3/8\pi$: this compares favorably to the free quark estimate of $\Gamma(c \to uq\bar{q})$. In b quark decays, $b\bar{q} \to s\bar{q}$ with intermediate t, c is not Cabibbo suppressed with respect to $b \to cq\bar{q}$. In $(b\bar{u})$ meson decays, $|k^2| \simeq m_u m_b \ll m_{c,t}^2$ and $F(k^2) \simeq (g_s/16\pi^2)(1/6)\log(m_t^2/m_c^2)$, suggesting $\Gamma(b\bar{u} \to$ $\to s\bar{u}) \simeq (\alpha_s/24\pi)^2\log^2(m_t^2/m_c^2)G_F^2(\sin\theta\cos\theta)^2[f_B^0]^2 m_b^3/8\pi$: presumably $[f_B^0]^2 \sim \Lambda^3/m_b$, so that (for $m_t < m_W$) this rate is small compared to $\Gamma(b \to cq\bar{q})$ from Fig. 4(b), and Fig. 4(c) is probably unimportant.

The estimates of Fig. 4(c) given above were all to lowest order in α_s: I now discuss the higher order corrections they receive. Consider, to be definite, the example of $s\bar{q} \to d\bar{q}$ with intermediate u or c. If $m_c = m_u$ (and $\bar{q} \neq \bar{u}$ or $\bar{c}$) then QCD corrections to the c and u exchange contributions must be identical: the mixing angles yield opposing signs for them, so that they cancel to all orders in α_s (but lowest order in G_F). If, for example, $\bar{q} = \bar{u}$, then in higher orders, there exist diagrams involving annihilation of the intermediate u with the spectator $\bar{u}$ (corresponding to the same amplitudes as "box diagram" corrections to Fig. 4(b)), which destroy cancellation between the u and c contributions. The local four-fermion form for the $s\bar{q} \to d\bar{q}$ interaction in Fig. 4(c) remains unmodified by higher order corrections: gauge invariance prevents the appearance of nonlocal terms involving exchanges with uncanceled $1/k^2$ propagators [F39]. As for Fig. 4(b), however, infrared divergences remain even after summing over all possible real and virtual gluon corrections with $m_{\bar{q}} \to 0$ [F40]. As mentioned above, at $0(g_s)$, $0(\log(m_W/m_Q))$ terms from Fig. 4(c) are always canceled by the GIM mechanism to $0(\log(m_{q_1'}/m_{q_2'}))$ ones. Any $0(\log(m_W/m_Q))$ corrections appearing in higher orders must usually have coefficients which vanish in the exact GIM limit $m_{q_1'} = m_{q_2'}$. It seems probable that the GIM cancellation removes all ultraviolet divergences which potentially occur as $m_W \to \infty$, so that only $0(\log(m_{q_1'}/m_{q_2'}))$ logarithmic factors appear in higher orders (as indicated by explicit $0(g_s^3)$ calculations [47]). As mentioned for $Q \to q''G$ above, the intermediate q' may interact with the "external field" present in the original meson: this field is presumably unable to absorb sufficient momentum for it to replace the perturbative G^* exchange.

Nonleptonic weak baryon decays should proceed by processes analogous to those for mesons in Fig. 4 [F41]. The independent Q decay mechanism of Fig. 4(a) (and of Fig. 4(c) if $Q \to q''G$ dominates) should have the same characteristics as in mesons. The W exchange diagram in Fig. 4(b1) induces the reactions $su \to ud$, $c(d, s) \to su$, $b(u, c) \to$ $\to c(d, s)$, etc., but does not occur with ss, sd, cu, etc., initial states. When these processes are embedded in baryons, the initial qq may have total angular momentum 0 or 1, usually with roughly equal probabilities. The helicity suppression encountered for $q\bar{q} \to q\bar{q}$ processes from Fig. 4(b) in spin 0 mesons is therefore absent for $qq \to qq$ processes in baryons. The single Q decay process analogous to Fig. 4(a) suggests that the weak decay rates of all baryons containing Q should be equal. The analogue of Fig. 4(b1) contributes only in baryons containing particular spectator quarks: its presence would, for example, imply different lifetimes for different baryons in an isomultiplet. The analogue of Fig. 4(c) with G^* exchange to a spectator q should behave in baryons much as in mesons. Since in baryons Fig. 4(b1) suffers no helicity suppression, naive estimates suggest that Fig. 4(c) is always smaller than Fig. 4(b1) whenever mixing angles allow the latter to contribute. If Fig. 4(a) and its analogues dominate all nonleptonic decays, then the lifetimes of baryons and of mesons containing a given heavy quark Q should be approximately equal. The absolute importance of Fig. 4(b) compared to Fig. 4(a) depends on $|\psi_{Qq}(0)|^2/m_Q^3$. Presumably $|\psi_{Qq}(0)|^2$ is similar to wavefunction at the origin which determines the magnitude of hyperfine splittings between, e.g., $J = 1/2$ and $J = 3/2$ baryons. In the nonrelativistic approximation, $|\psi_{Qq}(0)|^2 \sim [m_q^{eff}]^3$. For the decay of the charmed baryon $\Lambda_c(= cud)$, Fig. 4(b) may contribute; if $f_{\Lambda_c} \sim f_D$ then $\Gamma(\Lambda_c \to X) \sim \Gamma(D^0 \to X)$ [F42].

I now summarize the discussion of inclusive nonleptonic weak decays based on Fig. 4 given above, and relate it to some relevant experimental data. For hadrons containing t quarks ($m_t \gtrsim 18$ GeV) the independent $t \to bW^*$ decay process of Fig. 4(a) should be dominant, and the final states of the decays should consist of three resolvable hadron jets. (Even for $t\bar{b}$, helicity suppression renders Fig. 4(b) insignificant). Production of mesons containing b quarks close to threshold has recently been observed [48]: detailed data on their decay properties is not yet available, but will presumably soon be forthcoming. (Analysis of H_2 distributions nevertheless indicates the expected [31] spherical event structure [48] which was hinted at by higher energy data on inclusive production [49]). In these decays, Fig. 4(a) should again dominate. Mixing angles do not suppress Fig. 4(b) in $b\bar{d}$, $b\bar{s}$ and $b\bar{c}$ mesons: however, the smallness of $(f_B/m_B)^2 \sim |\psi_B(0)|^2/m_B^3 \sim (m_{\bar{q}}/m_b)^3$ prevents a substantial contribution except in the $b\bar{c}$ case, where the process $b\bar{c} \to \bar{c}s$ may be roughly comparable to $b \to cW^*$. Figure 4(c) can give $b\bar{q} \to s\bar{q}$ without mixing angle suppression (relative to $b \to cW^*$): again, however, the decay mechanism is rendered insignificant by the fact that $f_B \ll m_B$, and by small numerical factors entering in the loop diagram. Assuming that Fig. 4(a) dominates, one expects a semileptonic branching ratio $\Gamma(b \to lX)/\Gamma(b \to X) \simeq 1/[1+2(1+\alpha_s/\pi)] \sim 1/3$ where $l = e, \mu, \tau$. If, as indirect phenomenological evidence suggests, $\Gamma(b \to uW^*)/\Gamma(b \to cW^*) \lesssim 0.1$ [50], $\Gamma(b \to cs\bar{c}) \sim$ $\sim \Gamma(b \to cd\bar{u})$, so that about half of all hadronic b decay final states potentially contain two charmed hadrons. With Fig. 4(a) dominant, the lifetime of mesons containing b quarks would be $\simeq 10^{-15}$ sec (corresponding to a track length ~ 5 μm at a laboratory energy ~ 100 GeV).

Several thousand decays of $D\bar{D}$ pairs produced near threshold (at $\psi(3.77)$) in e^+e^- annihilation have been analyzed [25]. (In a few years, the Mark III detector at SPEAR should collect some million such D decays, allowing a more precise phenomenological investigation). A few D decay events have been observed directly in (triggered/hybrid) emulsion experiments [51] (and more are expected in new high-resolution detectors (e.g., [52])). Several experiments observe D production in hadronic collisions through specific exclusive or inclusive decay channels: in most cases, these give no additional information on D decays. F production in e^+e^- annihilation still awaits confirmation [F43]; a few candidate F decay events were found in emulsion [51] and there is some indication of hadronic F production [54]. Charmed baryon Λ_c (cud) pair production has been observed in e^+e^- annihilation [53], but only with the specific decay channel $\Lambda_c \to pK^-\pi^+$. A few Λ_c candidates have been found in emulsion experiments [52], and there have been several measurements of Λ_c production in hadronic reactions (some indicating a rather large forward production cross-section). Nonleptonic D^+ decays should be dominated by Fig. 4(a); in nonleptonic D^0 decays Fig. 4(b1) may also contribute, and naive estimates given above indicate that it could well dominate over Fig. 4(a). Semileptonic D^+ and D^0 decays should proceed through Fig. 3(a). Thus the semileptonic branching ratio $\Gamma(D^+ \to lX)/\Gamma(D^+ \to X)$ should be $\simeq 2/[2+3(1+\alpha_s/\pi)] \simeq 40\%$, and the (proper) D^+ lifetime would be $\tau_{D^+} = 1/\Gamma(D^+ \to X) \sim \tau_\mu (m_\mu/m_c)^5 1/5 \sim 10^{-12}$ sec. Figure 3(a) implies $\Gamma(D^0 \to lX) = \Gamma(D^+ \to lX)$; Fig. 4(b1) can yield $\Gamma(D^0 \to X) \gg \Gamma(D^+ \to X)$, and hence $\tau_{D^0} \ll \tau_{D^+}$ and $\Gamma(D^0 \to lX)/\Gamma(D^0 \to X) \ll \Gamma(D^+ \to lX)/\Gamma(D^+ \to X)$. Experimental results from $D\bar{D}$ production in e^+e^- annihilation [25] give $\Gamma(D^+ \to eX)/\Gamma(D^+ \to X) = (16.8 \pm 6.4)\%$, $\Gamma(D^0 \to eX)/\Gamma(D^0 \to X) = (5.5 \pm 3.7)\%$, supporting the hypothesis that Fig. 4(a) dominates D^+ nonleptonic decay, and indicating a significant contribution from Fig. 4(b1) to D^0 nonleptonic decay. Assuming $\Gamma(D^0 \to lX) = \Gamma(D^+ \to lX)$ these experimental results suggest that $\Gamma(D^0 \to X)/\Gamma(D^+ \to X) \simeq 3.1 \pm 2.3$. Direct measurements of emulsion events give [51] $\tau_{D^+} = (10.0^{+9.2}_{-4.0}) \times 10^{-13}$ sec (5 events), $\tau_{D^0} = (1.03^{+0.56}_{-0.32}) \times 10^{-13}$ sec (7 events), $\tau_F = (2.14^{+2.7}_{-1.0}) \times 10^{-13}$ sec (2 events), and $\tau_{\Lambda_c} = (1.14^{+0.62}_{-0.36}) \times 10^{-13}$ sec (6 events), indicating that $\Gamma(D^0 \to X)/\Gamma(D^+ \to X) \simeq (9.7^{+10.4}_{-4.9})$. The near agreement of this result with that deduced from the semileptonic branching ratios indicates that the assumption $\Gamma(D^0 \to lX) = \Gamma(D^+ \to lX)$ is approximately correct. As discussed above, nonleptonic F decays may receive contributions from Figs. 4(a) and 4(b2): the $u\bar{d}$ final state in this case should make the helicity suppression of Fig. 4(b) more effective (by a factor $\sim [(m_u^{eff}+m_d^{eff})/(m_s^{eff}+m_u^{eff})]^2$ which perhaps ~ 0.6) than in D^0 decay, yielding $\Gamma(F \to X)$ intermediate between $\Gamma(D^0 \to X)$ and $\Gamma(D^+ \to X)$, as suggested by the F lifetime measurement quoted above. Semileptonic decays $F \to lX$ should again be dominated by Fig. 3(a). Since $m_F \simeq 2$ GeV the decay $F \to \tau\nu_\tau$ may also occur: if indeed Fig. 4(b) dominates nonleptonic F decay, the naive estimate of it given above implies $\Gamma(F \to \tau\nu_\tau)/(F \to X) \sim \Gamma(F \to \tau\nu_\tau)/\Gamma(F \to u\bar{d}) \sim 1/3[m_\tau/(m_u^{eff}+m_d^{eff})]^2[1-(m_\tau/m_F)^2]^2 \sim 0.1$. (A direct estimate using the guess $f_F \simeq 0.5$ GeV and the "measured" F lifetime suggests a still larger branching ratio, perhaps ~ 0.3.) For the charmed baryon Λ_c, one expects significant contributions from the analogue of Fig. 4(b1), suggesting $\Gamma(\Lambda_c \to X) \sim \Gamma(D^0 \to X)$, in conflict with the experimental lifetime determination quoted above [51]. (This discrepancy would be removed if a large fraction of the experimental Λ_c candidates were, in fact, $\Sigma_c(= cuu)$, for which Fig. 4(b) can give no contribution, leaving Fig. 4(a) dominant, and yielding a lifetime comparable to that of D^+). An important consequence of Figs. 4(a) and 4(b) in D decays is that only a fraction $\simeq \tan^2\theta_c \sim 0.05$ of the final states should involve no s quark, and thus have $|S| = 0$. Important contributions from Fig. 4(c) (which are not expected according to the naive estimates given above) would enhance this ratio. Experimentally, only the two body decay modes $D^0 \to \pi\pi$ and $D^0 \to \bar{K}K$ have been measured [25]; their rate relative to $D \to K\pi$ is roughly consistent with $\tan^2\theta_c$ (see below) [F44]. In F decays, Fig. 4(a) should give predominantly $F \to \bar{s}su\bar{d}$, presumably yielding hadron final states containing ϕ or $\bar{K}K$, while Fig. 4(b) would give $F \to u\bar{d}$, yielding hadron final states containing η, π, but much fewer ϕ or $\bar{K}K$. The scanty experimental data on F production do not yet allow discrimination between these two cases [F45]. The final states of Λ_c decays experimentally appear to be predominantly $|S| = 1$, as expected from Fig. 4.

For strange particle decays, the inclusive treatment of decay rates given above is largely inappropriate: the energy released in the decays is so low that the detailed types and masses of the final hadrons are crucial. Approximating the final quarks in K weak decays as free, one expects $\Gamma(K^+) \simeq \Gamma(K_S^0) \simeq \Gamma(K_L^0)$ [F46]: in practice $\Gamma(K^+) \simeq 7\times10^{-3}\Gamma(K_S^0) \simeq \simeq 4\Gamma(K_L^0)$. The origin of this failure is clarified by consideration of the partial decay widths to particular pion final states. Experimental results show that $\Gamma(K^+ \to \pi e\nu) \simeq 2\Gamma(K_L^0 \to \to \pi e\nu)$; $\Gamma(K_L^0 \to \pi\pi\pi) \simeq 1.1\Gamma(K^+ \to \pi\pi\pi) \simeq 1.5\Gamma(K^+ \to \pi e\nu)$. Taking a matrix element uniform in the available phase space (which is found experimentally to be a reasonable approximation) suggests $\Gamma(K \to \pi\pi\pi) \sim 0.06\Gamma(K \to \pi e\nu)$: the actual rate for $K \to \pi\pi\pi$ is enhanced by a factor $\sim$ 15–30 over this estimate. A uniform matrix element (scaled by m_K) suggests [F47] $\Gamma(K \to \pi\pi) \sim 200\Gamma(K \to \pi e\nu)$ (e.g., [56]): in practice, $\Gamma(K^+ \to \to \pi^+\pi^0) \simeq 4\Gamma(K^+ \to \pi e\nu)$ while $\Gamma(K_S^0 \to \pi\pi) \simeq 3\times10^3\ \Gamma(K^+ \to \pi e\nu)$. The difference between semileptonic and nonleptonic decay rates even after accounting for phase space effects would result (as in the case of D mesons) from contributions of Figs. 4(b) and 4(c) as well as 4(a). The suppression of $K^+ \to \pi\pi$ compared to $K_S^0 \to \pi\pi$ by a factor $\sim 1/700$ indicates that the effective nonleptonic weak Hamiltonian for K decays transforms under strong isospin predominantly as $|\Delta I| = 1/2$. In $K^+ \to \pi^+\pi^0$, the final $\pi^+\pi^0$ must have $I = 2$, while in $K^0 \to \pi\pi$, they may have $I = 0$: the ratio of $\Gamma(K^+ \to \pi^+\pi^0)$ to $\Gamma(K^0 \to \pi\pi)$ is thus explained if the effective weak Hamiltonian responsible is predominantly $|\Delta I| = = 1/2$, with only $\sim 4\%$ $|\Delta I| = 3/2$ (or $|\Delta I| = 5/2$) component. The basic $s \to uW^*$ weak vertex has $|\Delta I| = 1/2$, while $u \to dW^*$ involves $|\Delta I| = 1$. In the free quark approximation, the $s \to u\bar{u}d$ and $s\bar{u} \to d\bar{u}$, $s\bar{d} \to u\bar{u}$ processes of Fig. 4(a, b) contain $|\Delta I| = 1/2$ and $|\Delta I| = = 3/2$ terms (but no $|\Delta I| = 5/2$ component) in equal concentrations (up to Clebsch-Gordan coefficients). It will turn out (see below) that the different isospin components of these reactions correspond to different color components: perturbative QCD corrections thus affect the relative concentrations of the two isospin components (probably enhancing $|\Delta I| = 1/2$). The process $s \to dG^*$ of Fig. 4(c) is purely $|\Delta I| = 1/2$. Note that in the present considerations of exclusive hadron final states, the diagrams of Fig. 4 must be added coherently, and the processes illustrated there may interfere. Only quark subprocesses with the correct isospin transformation properties can contribute in hadronic decays to hadron final states of definite isospin. Note, however, that isospin invariance is respected in the development of the hadron final state only inasmuch as the u and d are indistinguishable: electromagnetic interactions and $m_u - m_d \neq 0$ effects may modify the isospin of the final state. In hadronic terms, a virtual η may be produced initially, and may then mix through $|\Delta I| = 1$ isospin violation (such as is responsible for $\eta \to \pi^* \to \pi\pi\pi$ decay) to π with an amplitude at the 0 (1%) level [57]. Such effects could almost account for $\Gamma(K^+ \to \pi^+\pi^0)$ even if H_{weak} were pure $|\Delta I| = 1/2$: they should also lead to $0(10^{-3}$–$10^{-4})$ $|\Delta I| = 5/2$ contributions. The hypothesis that H_{weak} transforms approximately as $|\Delta I| = 1/2$ yields relations between rates for other K decays. A $\pi\pi$ final state may have $I = 0$ or $I = 2$: the $|\Delta I| = 1/2$ rule requires $I = 0$. Writing the amplitude for $|\Delta I| = J$ transitions as A_J, one has $\Gamma(K^+ \to \pi^+\pi^0)/\Gamma(K_s^0 \to \pi\pi) = 3/4 \; |A_{3/2}/A_{1/2}|^2 \simeq 1.5 \times 10^{-3}$, implying $|A_{3/2}/A_{1/2}| \sim 0.05$. In $K_s^0 \to \pi\pi$, both $|\Delta I| = 1/2$ and $|\Delta I| = 3/2$ transitions may occur, and may interfere. One finds in this case (ignoring 0(1%) phase space corrections) $\Gamma(K_s^0 \to \pi^+\pi^-)/\Gamma(K_s^0 \to \pi^0\pi^0) \simeq 2[1 + 3\sqrt{2}\ \mathrm{Re}[A_{3/2}/A_{1/2}]\cos(\delta_2 - \delta_0)]$ where δ_I is the strong interaction final state phase shift for a $\pi\pi$ system with isospin I, and $\delta_2 - \delta_0 \sim -50°$ [F48], [58]. Experimentally $\Gamma(K_s^0 \to \pi^+\pi^-)/\Gamma(K_s^0 \to \pi^0\pi^0) \simeq (2.19 \pm 0.01)$, again implying $|A_{3/2}/A_{1/2}| \sim 0.05$. Thus there appears to be a universal $|\Delta I| = 3/2$ contribution to $K \to \pi\pi$ decays, with an amplitude $\sim 1/20$ that of the dominant $|\Delta I| = 1/2$ term. Assuming that any $|\Delta I| = 5/2$ contribution is small, the experimental $\Gamma(K \to \pi\pi)$ imply that its amplitude $\leqslant 3 \times 10^{-3}$ that of the $|\Delta I| = 1/2$ term. $\pi\pi\pi$ final states may have $I = 0$, 1, 2, 3 ($I = 0$, 2 are probably much suppressed by centrifugal barrier effects, since they must involve $l = 1$): the $|\Delta I| = 1/2$ part of the weak Hamiltonian gives only $I = 0$, 1, while $I = 3$ can be reached only by $|\Delta I| = 5/2, 7/2$. If $|\Delta I| > 3/2$ terms are absent, $\Gamma(K^+ \to \pi^+\pi^+\pi^-) \simeq 4\Gamma(K^+ \to \pi^+\pi^0\pi^0)$, $\Gamma(K_L^0 \to \pi^0\pi^0\pi^0) \simeq 3/2\Gamma(K_L^0 \to \pi^+\pi^-\pi^0)$. These relations receive corrections from final state isospin violation effects. Two decays with equal amplitudes uniform in the available phase space have widths $\Gamma(K \to \pi\pi\pi) \sim (m_K - \Sigma m_\pi)^2/m_K$, which differ by virtue of $m_{\pi^+} \neq m_{\pi^0}$, $m_{K^+} \neq m_{K^0}$. Taking such a phase space correstion in $K^+ \to \pi^+\pi^+\pi^-$ gives a correction $\simeq 1.25$ to the relation $\Gamma(K^+ \to \pi^+\pi^+\pi^-) = 4\Gamma(K^+ \to \pi^+\pi^0\pi^0)$. If instead, the measured phase distribution is used, the correction $\simeq 1.17$. (Final state Coulomb interactions between $\pi^\pm$ provide a further $\sim 4\%$ correction). Experimentally, dividing out all kinematic isospin violation [60], $\Gamma'(K^+ \to \pi^+\pi^+\pi^-) \simeq (3.7 \pm 0.1)\Gamma'(K^+ \to \pi^+\pi^0\pi^0)$, $\Gamma'(K_L^0 \to \pi^0\pi^0\pi^0) \simeq (1.55 \pm 0.07)\Gamma'(K_L^0 \to \pi^+\pi^-\pi^0)$: uncertainties in removal of final state isospin violation effects preclude definite conclusion of $|\Delta I| > 3/2$ terms in $K^+ \to \pi\pi\pi$. Ignoring final state isospin violation, $\Gamma(K_L^0 \to \pi^+\pi^-\pi^0)/\Gamma(K^+ \to \pi^+\pi^0\pi^0) \simeq 2[1 - 3\ \mathrm{Re}[A_{3/2}/A_{1/2}]]$ (both final states are dominantly $I = 1$, so that no phase shift difference enters). Experimentally, $\Gamma'(K_L^0 \to \pi^+\pi^-\pi^0)/\Gamma'(K^+ \to \pi^+\pi^0\pi^0) \simeq (1.65 \pm 0.15)$, probably indicating $|A_{3/2}/A_{1/2}| \sim 0.06$, in rough agreement with the $|\Delta I| = 3/2$ contribution deduced for $K \to \pi\pi$. Further measures of $|\Delta I| > 1/2$ terms involving the distribution of final π energies in $K \to \pi\pi\pi$ also indicate similar concentrations, but are severely hampered by kinematic isospin violations. Of four particle K decay modes, only $K^+ \to \pi\pi e\nu$ has been measured. Assuming a matrix element uniform in available phase space (and scaled by m_K) suggests $\Gamma(K^+ \to \pi\pi e\nu)/\Gamma(K^+ \to \pi e\nu) \sim 1/(96\pi^2) \sim 10^{-3}$;

experimentally, $\Gamma(K^+ \to \pi\pi e\nu)/\Gamma(K^+ \to \pi e\nu) \simeq (1.2\pm0.7)\times10^{-3}$. The relative strengths of semileptonic and nonleptonic $|\Delta I| = 1/2$ and $|\Delta I| = 3/2$ contributions to K decays thus appear to be roughly independent of the specific decay considered, and to have amplitudes in the ratio $|A_{sl}|:|A_{1/2}|:|A_{3/2}| \sim 1:3:0.15$.

Semileptonic K decay presumably occurs basically through the mechanism of Fig. 3(a). However, since the energy released is small, the final hadron system usually consists only of a single pion, with a definite isospin $|I| = 1$. To form this pion, the final u from $s \to ul\nu$ and the spectator $\bar{u}$, $\bar{d}$ must be in an $|I| = 1$ rather than $|I| = 0$ state. In the case of K_L^0 decay, the $u\bar{d}$ must have $|I| = 1$. In K^+ decay, the $u\bar{u}$ initially have equal amplitudes $1/\sqrt{2}$ to be in an $|I| = 1$ or an $|I| = 0$ state. Because of the small energy release, hadronic effects force the $u\bar{u}$ to have $|I| = 1$, and thus suppress $\Gamma(K^+ \to \pi e\nu)$ with respect to $\Gamma(K^0 \to \to \pi e\nu)$ by a factor 2. (If m_K were larger, so that many π were produced in K decays, the $u\bar{u}$ could have either isospin with eventually equal amplitudes, and the rates for semileptonic K^0 and K^+ decay would become equal.) In nonleptonic K decays, the small number of final pions again introduces contraints on the total isospin of the quark systems from which they form. The total amplitude for a K decay may be considered roughly as a product of the amplitude for a quark system with particular isospin to be produced and the amplitude for that system to form the final pions, summed over all possible isospin states. Because of the isospin invariance of strong interactions, there exist direct relations between both amplitudes for systems of specific $|I|$ and different I_3, as exploited above. In most cases, comparisons of either amplitude for different $|I|$ are difficult. (An exception is the example of $K^+ \to \pi^+\pi^0$ (see above) where symmetries do not allow an $|I| = 0$ $\pi\pi$ state, so that an $|I| = 0$ quark system has zero amplitude to transform into the final state and induce the decay). Even after the final π have been "produced", they still undergo strong final state interactions, which may modify the amplitudes for different $|I|$. Interactions between outgoing on-shell final pions have an amplitude (by unitarity) of unit modulus. Because the pions propagate on-shell between successive rescatterings (see [F48]), the amplitude attains an imaginary part proportional to the rescattering amplitude, and gives a phase $e^{i\delta_I}$ to the amplitude for a decay with final state isospin $|I|$. This phase difference between the amplitudes for $K^+ \to \pi^+\pi^0$ and $K^0 \to \pi\pi$ was accounted for in the comparison of these processes above. However, in addition to these pure phase factors arising from interactions between on-shell outgoing pions, the modulus of the total decay amplitude is modified by final state interactions acting between off-shell pions: these distort the usual outgoing plane waves and alter the "wavefunction at the origin" for the $\pi\pi$ system [F49]. A quantitative estimate of such effects is very difficult: results are inevitably sensitive to the composite structure of the pion, and it is imposible to disentangle effects of "final state interactions" from features of the "primary interaction". Nevertheless, there are some qualitative indications that "final state" $\pi\pi$ interactions should enhance the rate for production of $|I| = 0$ over $|I| = 2\pi\pi$ systems. At low energies (below $K\bar{K}$ threshold) the relevant s-wave $\pi\pi$ elastic scattering phase shifts are well-fit by a scattering length approximation $\delta_I \simeq a_I k$; in the (nonexotic) $I = 0$ channel the $\pi\pi$ interaction is strongly attractive, with $a_0 \sim 0.5$ GeV^{-1}, while in the (exotic) $|I| = 2$ channel, it is slightly repulsive, with $a_2 \sim -0.05$ GeV^{-1} [58]. Final state $\pi\pi$ interactions should thus tend to enhance $|\Delta I| = 1/2$ ($I = 0$ final state) processes relative to $|\Delta I| = 3/2$ ones. (Attempts to obtain a quantitative estimate of this effect from, e.g., a comparison of the factors $\left|\exp\left[\int_0^\infty \frac{ds}{\pi}\,\delta(s)/(s-m_K^2)\right]\right|^2$, are thwarted by sensitivity to high s behavior, where unknown inelastic contributions are presumably important. A very rash guess is provided by $|\exp[(\delta_0(m_K^2)-\delta_2(m_K^2))/\pi]|^2 \sim 1.6$: not a large factor compared to the observed ratio ~ 400 of $|\Delta I| = 1/2$ to $|\Delta I| = 3/2$ rates).

In addition to such "large distance" effects, "short distance" phenomena, best considered in the framework of the quark diagrams Fig. 4, may also contribute to the suppression of $|\Delta I| = 3/2$ relative to $|\Delta I| = 1/2$ processes. Recall that the simple comparisons between measured K decay rates discussed above indicated that the ratio of semileptonic to nonleptonic decay amplitudes $|A_{sl}|:|A_{1/2}|:|A_{3/2}| \sim 1:3:0.15$. The ratio $|A_{1/2}|/|A_{3/2}|$ here may be obtained directly, e.g., from $\Gamma(K^+ \to \pi^+\pi^0)/\Gamma(K^0 \to \pi\pi)$. The deduction of the relative size of semileptonic and nonleptonic amplitudes requires some assumptions regarding the phase space structure of the decay rates: it remains possible (although unlikely) that the relevant semileptonic amplitude $|A_{sl}| \sim |A_{3/2}|$. Figures 4(a, b) contain both $|\Delta I| = 1/2$ and $|\Delta I| = 3/2$ components; the process $s \to dG^*$ of Fig. 4(c) is, however, pure $|\Delta I| = 1/2$. The simple free-quark estimate for $s\bar{q} \to d\bar{q}$ from Fig. 4(c) given above suggested that numerical factors associated with loop integration render it slightly smaller than Fig. 4(a). A serious quantitative estimate would, however, require greater information on the structure of hadrons than is yet available: it is still certainly conceivable that $|A_{1/2}| \gg |A_{3/2}| \sim |A_{sl}|$, with $|A_{3/2}|$ and $|A_{sl}|$ dominated by Fig. 4(a), and $|A_{1/2}|$ dominated by a larger term from Fig. 4(c). In the free quark approximation, the processes $s \to u\bar{u}d$ of Fig. 4(a) and $s\bar{u} \to d\bar{u}$ or $s\bar{d} \to u\bar{u}$ of Fig. 4(b) give essentially equal $|\Delta I| = 1/2$ and $|\Delta I| = 3/2$ amplitudes. The different isospin channels correspond to amplitudes with different symmetries under $u \leftrightarrow d$ interchange: the overall symmetry of the amplitudes then requires quark pairs in different SU(3) color representations. Thus gluon exchange corrections depend on the isospin properties of the amplitude, and may enhance $|\Delta I| = 1/2$ relative to $|\Delta I| = 3/2$ parts [F50]. Consider at first, for simplicity, the weak reaction $su \to ud$: this is directly relevant in nonleptonic weak hyperon decays; results for the cases of Fig. 4(a, b) will be obtained by crossing. The process $su \to ud$ proceeds at lowest order by t-channel W exchange: it receives corrections from real gluon emission and virtual gluon exchanges. The real gluon emission terms introduce much infrared divergence, but (at $O(\alpha_s)$) exhibit no ultraviolet divergences as $m_W \to \infty$, and thus can generate no $\log(m_W)$ terms. Virtual gluon exchanges do involve ultraviolet divergences, and thus may produce $O(\alpha_s \log(m_W))$ terms. Nevertheless, when all possible color quantum numbers of the initial and final qq states are averaged over, these ters cancel, as discussed above. However, if the color quantum numbers are restricted by requiring a specific isospin state, the $O(\alpha_s \log(m_W))$ terms no longer cancel, and serve to enhance the rates for production of some isospin states at the cost of others. At the energies $\ll m_W$ of concern here, the reaction $su \to ud$ must occur with essentially zero impact parameter, and thus involve no orbital angular momentum. The $V-A$ nature of the W coupling requires the interacting qq to have oppositely-directed helicities, so that the $su \to ud$ reaction occurs in a total angular momentum $J = 0$ channel, so that the spatial and spin parts of the final state wavefunction are antisymmetric under the interchange $u \leftrightarrow d$. The initial su state clearly has (strong) isospin $|I_i| = 1/2$. The final ud state may have $|I_f| = 0, 1$: if $|I_f| = 0$, then the complete $su \to ud$ reaction is purely $|\Delta I| = 1/2$; if $|I_f| = 1$, then it may involve a $|\Delta I| = 3/2$ component. When $|I_f| = 0$ the isospin part of the final ud wavefunction is antisymmetric under the interchange $u \leftrightarrow d$ $(\psi_0 \sim (ud-du)/\sqrt{2})$; when $|I_f| = 1$, it is symmetric $(\psi_1 \sim \sim (ud+du)/\sqrt{2})$. Assuming that the final ud obey Fermi-Dirac statistics, their total wavefunction must be antisymmetric under $u \leftrightarrow d$ [F51]. Thus if $|I_f| = 0$, the ud must be antisymmetric in their color quantum numbers, while if $|I_f| = 1$, they must be symmetric. The initial and final qq may transform under $SU(3)_c$ according to the representations $3 \otimes 3 = \bar{3} \otimes 6$: the $\bar{3}$ representation is antisymmetric in the quark indices, while the 6 is symmetric. (For $SU(N)_c$, the possible representations are $N \otimes N = \overline{[N(N-1)/2]} \otimes \overline{[N(N+1)/2]} \equiv D_A + D_S$, which are respectively antisymmetric and symmetric). $A |\Delta I| = 3/2$ $su \to ud$ reaction thus requires the initial and final qq to transform according to the symmetric, 6 representation of $SU(3)_c$; when $|\Delta I| = 1/2$, the qq may also transform under the antisymmetric $\bar{3}$ representation [F52]. The amplitude for virtual gluon exchange corrections to $su \to ud$ depends on the qq color representation: it will turn out that one gluon exchange is attractive (leading to an enhanced scattering amplitude) for the $\bar{3}$ representation, and repulsive for the 6. The (averaged) amplitude for one gluon exchange between qq in a color $SU(N)_c$ symmetric state is proportional to $W_S = (1-1/N)$ (the $1/N$ accounts for the absence of colorless gluons); in a color antisymmetric state, the amplitude is proportional instead to $W_A = (-1-1/N)$ [F53]. For the $SU(3)_c$ case, $W_S = 2/3$ while $W_A = -4/3$: one gluon exchange yields $0(\alpha_s \log(m_W/\mu))$ terms which enhance the $|\Delta I| = 1/2$ amplitude for $su \to ud$, and suppress the $|\Delta I| = 3/2$ amplitude. (Note that summing over all possible initial and final qq colors yields the required vanishing coefficient $W_S D_S + W_A D_A = 0$ for $\alpha_s \log(m_W/\mu)$). $0(\alpha_s \log(m_W/\mu))$ terms are not the only part of the one-gluon exchange amplitude which may depend on the qq color representation. Soft gluon emission and exchange occur coherently from the two quarks, and are thus potentially very sensitive to their total color. However, as mentioned above, QCD processes occurring at distances $\sim 1/\Lambda$ presumably neutralize the qq color, but do not affect their isospin (although they may modify the amplitudes for different $|I_f|$, e.g., through the "final state interactions" discussed in the previous paragraph). The connection between the qq isospin and color derived above holds only at short distances: at larger distances, one must account for gluon radiation; it seems probable that no significant dependence of the qq scattering amplitude on the original qq color (and isospin) will survive. Thus $0(\alpha_s \log(m_W/\mu))$ terms may plausibly be the only component of the $su \to ud$ amplitude which depend significantly on $|I_f|$. The discussion above indicates that the relevant infrared cutoff μ on the one-gluon exchange amplitude $\sim \Lambda$ (or the inverse size of the initial meson). In the simplest approximation, one may consider a sequence of independent gluon exchanges between the incoming and outgoing qq. The invariant masses of the exchanged gluons are as usual kinematically constrained to be ordered. The maximum invariant mass of the gluon closest to the W exchange is $\sim m_W$: for larger invariant masses the W exchange would cease to act as a point interaction, and the amplitude would be damped. The amplitude for n gluon exchange then $(\alpha_s(t) \sim 1/\log(t/\Lambda^2)) \sim \int_{\mu^2}^{t_2} dt_1/t_1\,\alpha_s(t_1) \int_{\mu^2}^{t_3} dt_2/t_2\,\alpha_s(t_2) \ldots \int_{\mu^2}^{m_W^2} dt_n/t_n\,\alpha_s(t_n)[\gamma]^n$ $\sim [\log\log(m_W^2/\Lambda^2)/\log\log(\mu^2/\Lambda^2)]^n[\gamma]^n/n!$, where γ is proportional to the color factors derived above, and accounts for integration over longitudinal kinematic parameters for the exchanges (cf., e.g., [66])). Summing the contributions from all possible numbers of exchanged gluons [F54] then gives a correction factor $\sim [\alpha_s(m_W^2)/\alpha_s(\mu^2)]^{-9W_I/(33-2F)}$: this suggests that the $|\Delta I| = 3/2$ $su \to ud$ amplitude is suppressed with respect to the $|\Delta I| = 1/2$ amplitude by a factor $\sim [\alpha_s(m_W^2)/\alpha_s(\mu^2)]^{9/(33-2F)} \sim [\alpha_s(m_W^2)/\alpha_s(\mu^2)]^{0.4}$. Lack of knowledge regarding the infrared cutoff μ^2 prevents a satisfactory quantitative conclusion from this result. Taking $\alpha_s(\mu^2) \sim 1$, it suggests $|A_{3/2}(su \to ud)|/|A_{1/2}(su \to ud)| \sim 0.3$–$0.5$. This estimate was based solely on a leading log approximation in which successive gluon exchanges are assumed (statistically) independent. To improve the approximation one must account for interference between successive emissions. The color factors for the corresponding diagrams (which involve, e.g., crossed gluon "rungs") exhibit no simple behavior in the relevant color symmetric and antisymmetric channels (even in the limit $N_c \to \infty$): an explicit calculation of all contributing diagrams is thus required [F55]. Having considered the process $su \to ud$, it is a matter of crossing to apply the results to the processes $s \to u\bar{u}d$ and $s\bar{d} \to u\bar{u}$, $s\bar{u} \to d\bar{u}$ of Figs. 4(a, b): perturbative QCD corrections should again provide some enhancement of $|\Delta I| = 1/2$ over $|\Delta I| = 3/2$ terms.

It is at present not possible to make a convincing quantitative conclusion on the origin of the isospin dependence of K decay rates. Three qualitative phenomena nevertheless suggest effects in the observed direction, but each alone is probably not of a sufficient magnitude. First, any contributions from Fig. 4(c) must be pure $|\Delta I| = 1/2$, and thus tend to enhance this component over $|\Delta I| = 3/2$ and semileptonic decays. Second, final state strong interactions in the $\pi\pi$ systems produced by the decays may depend on the final $\pi\pi$ isospin in such a way as to enhance $|\Delta I| = 1/2$tdecays relative to $|\Delta I| = 3/2$ ones. Third, gluon exchange effects at distances $\sim 1/m_W$ as estimated by a leading log approximation probably provide some enhancement of $|\Delta I| = 1/2$ over $|\Delta I| = 3/2$ amplitudes.

Weak hyperon decays in many respects parallel kaon decays. The only nonleptonic baryon decays (except $\Omega^- \to \Xi\pi\pi$), allowed by phase space constraints are $B \to B'\pi$, while of semileptonic decays, only $B \to B'l\nu$ have been observed. No meaningful quantitative conclusions on the relative sizes of the nonleptonic and semileptonic decay amplitudes may be drawn from comparisons between the measured rates for the two-body decays $B \to B'\pi$ and the three-body decays $B \to B'l\nu$ (very naive estimates based on matrix elements uniform in available phase space and scaled by m_B indicate that the amplitudes are equal to within an order of magnitude, with that for nonleptonic decays often the smaller). Semileptonic hyperon decays are presumably dominated by the process of Fig. 3(a): their rates are roughly independent of the types of the "spectator" quarks in the initial and final baryons (except through the initial and final wavefunction factors). The amplitudes for nonleptonic $B \to B'\pi$ decays are of the form $\bar{B}[\alpha+\beta\gamma_5]B'\pi$: the α term yields s-wave final states, while the β term gives p-wave ones, in which the final B' polarization is opposite to the initial B polarization. The assumption of a pure $|\Delta I| = 1/2$ effective weak Hamiltonian yields several relations between the various nonleptonic hyperon decay rates (which should hold separately for the s- and p-wave amplitudes). For the isoscalar hyperons Λ and Ω, the $|\Delta I| = 1/2$ assumption implies the relations $\Gamma(\Lambda \to p\pi^-) = 2\Gamma(\Lambda \to n\pi^0)$, $\Gamma(\Omega \to \Xi^0\pi^-) = 2\Gamma(\Omega \to \Xi^-\pi^0)$. Experimentally the first of these relations is valid to within $\sim 5\%$, while from the experimental measurement $\Gamma(\Omega \to \Xi^0\pi^-) \simeq (2.39 \pm 0.45)\Gamma(\Omega \to \Xi^-\pi^0)$ it appears that the second is considerably violated, implying a $|\Delta I| = 3/2$ amplitude in this case $|A_{3/2}|/|A_{1/2}| \sim 0.1$–$0.2$. Making the (perhaps questionable) assumption that the matrix elements of the effective weak Hamiltonian for baryons in the same isomultiplet are related by isospin rotations yields as a further consequence of the $|\Delta I| = 1/2$ transformation of the effective Hamiltonian the relations $A(\Sigma^+ \to n\pi^+) + \sqrt{2}\,A(\Sigma^+ \to p\pi^0) = A(\Sigma^- \to n\pi^-)$ and $\Gamma(\Xi^- \to \Lambda\pi^-) = 2\Gamma(\Xi^0 \to \Lambda\pi^0)$. Experimentally, these relations are also violated by at most $\sim 5\%$ in amplitude. ($\Xi \to \Sigma\pi$ decays are forbidden by energetic constraints). The Lee-Sugawara relation $A(\Lambda \to p\pi^-) + \sqrt{3}\,A(\Sigma^+ \to p\pi^0) = 2A(\Xi^- \to n\pi^-)$ requires the further assumption that the matrix elements of the effective weak Hamiltonian for Ξ states is related to that for Σ, Λ by an unbroken $SU(3)_{\text{flavor}}$ rotation. It is again found to be valid experimentally to better than 10% (or a factor of 2 for p-wave amplitudes). All nonleptonic hyperon decays may receive contributions from the processes $s \to u\bar{u}d$ (cf., Fig. 4(a)) and $sq \to dq$ (cf., Fig. 4(c)). The W exchange process $su \to ud$ (cf., Fig. 4(b1)) can give a significant contribution only if the initial baryon contains a ("valence") u quark. This mechanism may therefore contribute to Λ, Σ^+ and Ξ^0 decays, but not to Σ^-, Ξ^- or Ω^- decays. The observed validity of the "$|\Delta I| = 1/2$ rule" relations between Σ^+, Σ^- and Ξ^0, Ξ^- decay rates mentioned above indicates, however, that this mechanism is probably not important, despite encouraging estimates made above (and, e.g., [68])). Just as in K decays, the isospin transformation properties of the effective weak Hamiltonian for hyperon decays should be affected by both final state hadronic interactions and by short distance perturbative QCD effects. The phase of the $B \to B'\pi$ amplitudes may again be determined by the $B'\pi$ elastic scattering phase shifts in the relevant s- or p-wave orbital angular momentum state. The modification to the modulus of the $B \to B'\pi$ amplitude due to final state hadronic effects is incalculable as in the K decay case. Qualitatively, however, it seems likely that $B \to B'\pi$ will be enhanced by such effects when the $B'\pi$ interaction is attractive at energies $\sim m_B$ (as revealed by the presence of resonances in the $B'\pi$ system). (For example, $|\Delta I| = 1/2$ dominance in Λ decays may in part be accounted for by the quantum numbers of resonances in the $N\pi$ system: the $I = 1/2$ N or even N^* (1470) pole is much closer to m_Λ than the lowest-lying $I = 3/2$, $J = 1/2$ resonance $\Delta(1650)$. However, Ω decays should be dominated by the $\Xi^*(1530)$ pole and thus predominantly $|\Delta I| = 1/2$, since there are no $S = -2$, $|I| \neq 1/2$ resonances, whereas experimental measurements mentioned above indicate a significant $|\Delta I| = 3/2$ component). The process $sq \to dq$ analogous to Fig. 4(c) is pure $|\Delta I| = 1/2$, and may be dominant in hyperon decays. The process $s \to u\bar{u}d$ (analogous to Fig. 4(a)) contains both $|\Delta I| = 1/2$ and $|\Delta I| = 3/2$ components, as does $su \to ud$ (cf., Fig. 4(b)). It was shown above that if the ud system in the final states of these latter processes is constrained to transform as a $\bar{3}$ representation under $SU(3)_c$, then the processes are pure $|\Delta I| = 1/2$. In the absence of color interactions (or gluons), this would be achieved if the final ud in $s \to u\bar{u}d$ or the initial su in $su \to ud$ were contained in a single baryon [69] [F56]. In practice, as discussed above, the colors of the initial (and final) quark states at large distances are probably irrelevent: they are arbitrarily modified by radiation of very soft gluons. Thus the enhancement of $|\Delta I| = 1/2$ over $|\Delta I| = 3/2$ amplitudes in hyperon decays should occur through the same effects, and to a roughly equal extent as in K decays.

The treatment of charmed meson decays given above was concerned primarily with purely inclusive final states. The rates for charmed meson decays to specific exclusive hadron final states presumably exhibit enhancements and suppressions analogous to those found in strange particle decays. The first important source of such modifications would be strong interactions between the final state hadrons. As in the case of K decays, one may estimate the relative phases of various decay amplitudes using Watson's theorem [70]: the probably important effect of final state interactions on the moduli of the decay amplitudes remains entirely incalculable. For two (and perhaps three) body final states, experimental phase shifts may be used to estimate the phases of the decay amplitudes [70]: the effects of these phases alone give numerically-important corrections. For weak decays of mesons with masses $m_M \gtrsim 2$ GeV, inelastic final state scatterings render Watson's theorem inapplicable to the phase of a particular decay amplitude: as $m_M \to \infty$, the random phases of the increasing number of contributing scattering amplitudes will presumably yield a decreasing phase difference between any two decay amplitudes. Just as in K decays, perturbative QCD effects at short distances may modify the rates for D decays to hadron final states with different transformation properties under interchanges of quark flavors. In the process $s \to u\bar{u}d$, gluon exchanges should enhance final states antisymmetric under the interchange $u \leftrightarrow d$, corresponding to $|\Delta I| = 1/2$; similarly, in $c \to su\bar{d}$ such effects should enhance final states antisymmetric under $s \leftrightarrow u$ compared to those symmetric in this interchange. The magnitude of this effect in D decays should be of the same order as in K decays: the infrared cutoff μ introduced above is determined by the inverse size of the initial meson, which is similar for the D and K cases (the effect may be slightly smaller in F than in D decays). If such short distance phenomena are the primary cause of $|\Delta I| =$ $= 1/2$ enhancement in K decays, then the enhancement of $s \leftrightarrow u$ antisymmetric over $s \leftrightarrow u$ symmetric final states in D decays should be by the same large factor. The $s \leftrightarrow u$ antisymmetric part of the effective Hamiltonian for D decay transforms as a *6* under u, d, s $SU(3)_F$ rotations (contained in *20* of $SU(4)_F$); the symmetric part transforms as a 15 under $SU(3)_F$ (contained in *84* of $SU(4)_F$) (e.g.,[71])). *6* dominance has many consequences for exclusive D decay rates (e.g., [72])). For example, all Cabibbo-favored two body decays (e.g., $D^+ \to \bar{K}^0\pi^+$) are forbidden. Experimentally (e.g., [25])), such decays are observed with branching ratios relative to three-body modes roughly as expected from available phase space. Further consequences of *6* dominance for differential widths in three-body decay modes await experimental investigation. It seems unlikely, however, that the suppression of $SU(3)_F$ *15* with respect to $SU(3)_F$ *6* amplitudes for D decays will be as marked as in K decays. If this is the case, then it indicates that perturbative QCD effects connected with symmetries between final state quarks are not particularly significant:

the dominance of the $|\Delta I| = 1/2$ component in K decays is thus presumably a consequence of important contributions from final state hadronic effects and Fig. 4(c).

Precise predictions even for inclusive decays of $Q\bar{q}$ (e.g., K, D) mesons are rendered impossible by the presence of the light $\bar{q}$ in the initial state, and the inevitable infrared divergences which accompany it. One case in which such difficulties are absent is for initial $Q\bar{Q}$ quarkonium states (e.g., ψ, Y; denoted generically ζ). Weak contributions in ζ decays may arise either through a t-channel W exchange process $Q\bar{Q} \to q\bar{q}$, or via an s-channel Z: $Q\bar{Q} \to Z^* \to q\bar{q}$. The presence of such weak amplitudes may be detected through the small parity-violating correlations which they induce [73] by interference with the dominant p-conserving electromagnetic or strong amplitude. In a free quark approximation, the weak amplitude for ζ decay is given roughly by $|A_{wk}(\zeta \to q\bar{q})|/|A_{em}(\zeta \to \to \gamma^* \to q\bar{q})| \sim (m_\zeta/m_{W,Z})^2$: for Y, this ratio is $0(2\%)$. When gluon exchange and emission effects are included, both weak and electromagnetic amplitudes are modified: however, as for Fig. 4(b) above, in the leading log approximation, no $0(\log m_W)$ terms appear in the weak amplitude, and the ratio suffers no large corrections. P violation may induce $\langle \vec{s}.\vec{p}\rangle \neq 0$ correlations, where $\vec{s}$ is a spin vector, and $\vec{p}$ is a three momentum. If longitudinally-polarized ζ are produced by collisions of longitudinally-polarized e^+e^- beams, then such correlations between the outgoing q (rather than $\bar{q}$) momentum $\vec{p}_q$ and the ζ spin may occur. (Correlactions between $\vec{p}_q$ and the incoming e^- spin are strictly not P-violating; however, the P-conserving $\gamma\gamma$ exchange background is negligible on resonance). The q momentum direction may be determined statistically by measurement of high-energy $\pi^\pm$. For Y, such P-violating weak effects should occur at the $0(10^{-3})$ level, and eventually be observable. For heavier (e.g., $t\bar{t}$) ζ states, the effects should be much larger.

I now comment briefly on radiative weak decays. These might occur either through single quark decay process such as $s \to d\gamma$, or as radiative corrections to the mechanisms of Fig. 4. As discussed above in connection with Fig. 4(c), however, the $s \to d\gamma$ vertex vanishes for on-shell photons (up to $0(m_Q^2/m_W^2)$): thus the single quark decay mechanism should not be important (it is found to be inadequate phenomenologically [74]). In radiative two-body hyperon decays such as $\Sigma^+ \to p\gamma$, it seems likely that the process $su \to \to du\gamma$ occurring as a radiative correction to Fig. 4(b) or 4(c) dominates: the γ must have energy $\sim m_s/2$, so that the ud are directed opposite to it into the same final baryon. The rate for this kinematic configuration is $0(\alpha/\pi)$ snd small compared to the pure $su \to du$ process (the relevant numerical coefficient has not yet been calculated, but is probably small). The fact that $\Gamma(\Sigma^+ \to p\gamma)/\Gamma(\Sigma^+ \to N\pi)$ is as large as $\simeq 10^{-3}$ is thus somewhat surprising. Nevertheless, final state hadronic effects are undoubtedly very important in such decays, and may largely determine, for example, the ratio of P-violating and P-conserving amplitudes (e.g., [75])). If radiative corrections to diagrams analogous to Fig. 4(c) dominate radiative hyperon decays, then the rates for all possible such decays should be similar. However, if radiative corrections to the analogue of Fig. 4(b) are important, then the radiative decay rates of Ξ^- and Ω^- should be smaller than those of Ξ^0 and Σ^+ (since Fig. 4(b) cannot contribute in the former baryons). The experimental result that $\Gamma(\Xi^- \to \Sigma^-\gamma)/\Gamma(\Xi^0 \to \Lambda\gamma) \lesssim 0.2$ perhaps suggests that the analogue of Fig. 4(b) is indeed important. Radiative decays involving more than two final particles (e.g., $K \to \pi\pi\gamma$) are usually dominated by Bremsstrahlung from the participating hadrons, and do not directly probe the weak interaction.

Acknowledgments

I am grateful to R. Baltrusaitis, J. Finjord, D. Hitlin, N. Isgur, G. Ross, R. Ruckl and particularly C. Hill for discussions.

FOOTNOTES

[F1] Recall that for free Dirac spinors $\psi_{1,2}$ such that $\partial\psi_{1,2} = m_{1,2}\psi_{1,2}$, $\partial_\mu(\overline{\psi}_1\gamma_\mu\psi_2) = (m_1 - m_2)\overline{\psi}_1\psi_2$, while $\partial_\eta(\overline{\psi}_1\gamma_\mu\gamma_5\,\psi_2) = (m_1 + m_2)\overline{\psi}_1\psi_2$.

[F2] For massless quarks, $\Gamma(\gamma^* \to X) = \Gamma(W^* \to X) \simeq \Gamma_0(1+\alpha_s(Q^2)/\pi + 0(\alpha_s^2))$.

[F3] The upper component of a Dirac spinor gives the amplitude for a fermion to populate a particular spin state; the lower component gives the amplitude for the opposite spin state ($\sim \vec{\sigma}\cdot\vec{q}/m$ in the nonrelativistic limit): in a bound state wavefunction, the opposite spin of lower components is compensated by orbital angular momentum (thereby, e.g., introducing p-wave components into a nominally s-wave hydrogen atom level).

[F4] The relation remains valid to all orders in QCD perturbation theory: it receives $0(G_{\mu\nu}\tilde{G}^{\mu\nu})$ and other corrections from non-perturbative effects.

[F5] In fact, for a potential of the form $V(r) = \lambda(r\lambda)^\xi$, $|\psi(0)|^2 \sim [\mu\lambda^{(1+\xi)}]^{3/(2+\xi)}$ (e.g., [9])) with $1/\mu = 1/m_1 + 1/m_2$.

[F6] That is, $f_M^2 m_M \sim \psi_1^+\psi_2^+ - \psi_1^-\psi_2^-$. For a non-relativistic pair with relative momentum $\vec{k}$, $\psi_1^-\psi_2^- \sim (\vec{k}^2/m_q^2)\psi_1^+\psi_2^+$. For ultrarelativistic systems, this probably becomes $\psi_1^-\psi_2^- \sim \psi_1^+\psi_2^+ + 0(m_M^2/\vec{k}^2)$ [11]. (I am grateful to C. Llewellyn Smith for a discussion on this matter).

[F7] These results were obtained by numerical solution of Schrodinger's equation, taking $m_s = 0.5$ GeV, $m_c = 1.5$ GeV and $m_b = 5$ GeV.

[F8] Box diagrams for $M \to l\nu_l$ in which a hard virtual photon accompanies the W^* apparently also suffer helicity suppression [14].

[F9] These effective magnetic moments would not exhibit the large isospin violation naively expected from $0(e/m)$ magnetic moments.

[F10] The Q charge determines whether W^+ or W^- are emitted in Q decays: the two possibilities involve conjugate quark currents, and exchange the q' and l spectra. The spectra given here assume $V-A$ $Qq'W^*$ couplings: for a $V+A$ coupling and "W^+" emission $1/\Gamma_0 d\Gamma_0/dx = x(2+3x/2-2x^2)$ (e.g., [18])). Note that three-body phase space alone gives $1/\Gamma_0 d\Gamma_0/dx = 2x$.

[F11] The exact form is given in Ref. [19]), but does not differ significantly from this approximation over the whole range of x. Note that the physical picture of gluon emisssions only from the outgoing q' is realized for the contributions of explicit Feynman diagrams in an axial gauge with $\eta\cdot\varepsilon_G = 0$ and $\eta \| \vec{p}_Q$.

[F12] This estimate goes far beyond the formal region of applicability of the double log approximation (2).

[F13] This estimate contains no explicit reference to the prescription used in renormalizing α_s, but assumes that $\mu^2 \simeq m_Q^2$. The complete $0(\alpha_s^2)$ form depends on renormalization prescription, and can be obtained only by explicit calculation.

[F14] As discussed above, current conservation (and hence the Ward identity) is violated by the unequal quark masses $m_Q \neq m_{q'}$: this violation is nevertheless too soft to affect ultraviolet behavior.

[F15] Such a fit is shown in Ref. [19] using the $0(\alpha_s)$ lepton spectrum; use of the higher order estimate (3) changes slightly the optimal parameters. Note, however, that the statistical quality of available data [25] is as yet quite insufficient to allow serious quantitative comparison.

[F16] Even if the initial Q, $\bar{q}$ were far off mass shell, the restriction to color singlet initial and final states forbids virtual corrections to one gluon emission and prevents cancellation of infrared divergences from radiation of an arbitrarily soft second gluon (which would cancel if no color restriction were imposed).

[F17] The formal infrared divergence of $0(\alpha_s)$ corrections to color magnetic moments is presumably regularized by the $\sim \Lambda^2$ color magnetic fields present in the M state [27].

[F18] That is, the diagrams of Fig. 3(a) and 3(b) must be added before squaring, and the interference between the two mechanisms included.

[F19] As for nucleons probed by spacelike invariant mass γ^*, the effective partitioning of energy between quark and gluon species in M depends on the invariant mass of the probe used to measure it. Typically, a large invariant mass μ probe effects a measurement in a short time $\sim 1/\mu$, during which the constituents may be off their mass shells by an amount $\lesssim \mu$: they thus radiate with probabilities $\sim \log(\mu/m)$. Here, however, $\mu \sim m_Q$, allowing little radiation, so that the W^* probes the "primordial" M wavefunction. (In the analogous case of non-leptonic decays discussed below, $\mu \sim$ $\sim m_W$, so that for $m_Q \ll m_W$ significant radiation may occur).

[F20] As elsewhere, I assume that no flavor-changing neutral currents are present. If M is a $Q\bar{Q}$ state, then in Fig. 4(b2), the intermediate W^* may be replaced by Z^{0*}.

[F21] The methods used to obtain the guess (2) for Fig. 3(a) are inapplicable here.

[F22] If the product momenta are not large enough, then there will also be significant back reactions with the initial Q, etc.: for small outgoing velocities v, these modify the total rate by a factor $\sim 1+$ $+2\pi\alpha_s/v+\ \ldots$.

[F23] With charge 2/3, $-1/3$ quarks, it is not possible for q' and q_1 in $Q \to q'q_1\bar{q}_2$ to have the same flavor. Note that all q produced by W interactions are left-handed, and are therefore in the same helicity state.

[F24] Statements to the contrary in the literature result from neglect of soft gluons from the initial M. That such gluons must be present may formally be seen from the fact that they are required to cancel infrared divergences associated with the massless incoming $\bar{q}$.

[F25] Assuming that only the "valence" $\bar{q}$ is present in a given M state. The large W mass samples the M at very short distances, at which significant "sea" $q\bar{q}$ pairs may be present: their effects will be mentioned below.

[F26] Only if the invariant mass of the W^* extracted from $|\psi(0)|^2$ were varied would calculable corrections to $|\psi(0)|^2$ result (as essentially occurs for the Drell-Yan process).

[F27] In the semileptonic case of Fig. 2(b), color conservation formally required two gluons to be emitted $((Q\bar{q}) \to GGW^*)$, although one of the gluons could be arbitrarily soft, and may be subsumed into the initial M wavefunction. Two gluon emission is also formally required in Fig. 4(b2). In Fig. 4(b1), however, color may be conserved with only one gluon emission. Nevertheless, as discussed for Fig. 2(b), the presence of initial soft gluons undoubtedly renders these differences irrelevant.

[F28] In this nonleptonic process, there exist further diagrams, not present for Fig. 2(b), in which gluons are exchanged in parallel with the W^*. As discussed above, these are absent at $0(\alpha_s)$. Analogy with $\gamma^* W^*$ "box diagram" contributions to $M \to l\nu$ suggests that they are always negligible.

[F29] This is a component of the manifestly gauge invariant form $\sim M\bar{s}\sigma_{\mu\nu}F^{\mu\nu}d$, where $F^{\mu\nu} = q^\mu A^\nu -$ $-q^\nu A^\mu + g_S[A^\mu, A^\nu]$ is the gluon field strength. This interaction also induces $0(g_S^2)s \to dGG$ decays with an amplitude related to the magnetic moment $s \to dG$ decays discussed here.

[F30] Since the M are taken to have spin 0, the directions of the M and $\bar{M}$ decay products are uncorrelated: the $\langle H_l \rangle$ therefore obey linear superposition, and $\langle H_l \rangle$ for an $M_1\bar{M}_2$ final state is given simply by [31] $\langle H_l \rangle = 1/4(\langle H_l^1 \rangle + \langle H_l^2 \rangle)$.

[F31] That this is finite and contains no $0(\log(m_W))$ terms is evident from the absence of a possible counterterm for it in the original Lagrangian. Note that $s \to dG$ could involve not only a magnetic dipole term $\sim \bar{s}\sigma_{\mu\nu}q^\mu\varepsilon^\nu d$, but also an electric dipole term $\sim \bar{s}\gamma_5\sigma_{\mu\nu}q^\mu\varepsilon^\nu d$. This is not T-violating (as

$\bar{\mu}\gamma_5\sigma_{\mu\nu}q^\mu\varepsilon^\nu\mu$ would be) because the s and d are distinct and have different masses. (Recall that while a nonzero static electric dipole moment requires T violation, transition electric dipole moments do not: hence, for example, a molecule can effectively have an electric dipole moment through electric dipole transitions between its closely-spaced rotational levels). The coefficient of the E1 term is constrained by equations of motion to be $-(m_s-m_d)/(m_s+m_d)$ times the M1 term: the ratio of the E1 to M1 terms determines the magnitude of P violation in a $s \to dG$ decay, and thus the angular correlation of the G momentum with respect to the initial s spin direction.

[F32] Recall that Fig. 4(c) yields $b \to sG$ while Fig. 4(a) gives $b \to cW^*$. Failure to observe $b \to sG$ at the 0(20%) level would provide an upper limit $\sim m_W$ on the t quark mass. The generation of m_t from spontaneous symmetry breakdown in the standard Weinberg-Salam model requires $m_t \lesssim \lesssim \sqrt{3}\, m_W$.

[F33] Similar effects may occur in the presence of instanton background fields. In an operator product expansion analysis they would presumably appear through operators (e.g., proportional to $F.\tilde{F}$) which vanish in perturbation theory.

[F34] This form is only valid for "m_c^2" $\ll m_W^2$: the contribution of intermediate quarks with $m_Q \gg m_W$ is presumably $0(m_Q/m_W)$.

[F35] The exact $F(k^2)$ may be written in the Feynman parametric form $(g_S/16\pi^2)\int_0^1 dx x(1-x)\log[[m_c^2- -q^2x(1-x)]/[m_u^2-q^2x(1-x)]]$.

[F36] The gauge invariant form of the $s \to dG$ *amplitude is $\bar{s}\gamma^\mu dDF^{\mu\nu}$, where $D_\nu = \partial_\nu - igG_\nu$ is a covariant derivative, and $F^{\mu\nu}$ is the gluon field strength tensor. The QCD equations of motion relate the amplitude for the virtual gluon to the amplitude for its source: $D_\nu F^{\mu\nu} = g_S J^\mu = g_S \sum_i \bar{q}_i\gamma_\mu q_i$. (These equations of motion are respected to all orders in ordinary perturbation theory; they are modified by semi-classical (e.g., instanton) effects). Thus $s \to dG^*$ may always be considered as $\bar{q}s \to d\bar{q}$ or $s \to dq\bar{q}$. Note that processes such as $s \to dGG$, where the virtual G^* "decays" into GG, do not occur at this order.

[F37] This conclusion may also be reached by applying the equation of motion $\bar{q}_L q_R = q_L \partial\bar{q}_L/(m_q+m_q)$ (cf., [F1]) [42].

[F38] The complete formula for the exchanged k^2 in the process $ab \to cd$ with a, b at rest is $k^2 = (m_a m_a^2 + +m_b m_c^2 - m_a m_b^2 - m_a^2 m_b)/(m_a+m_b)$ $(\leqslant 0)$.

[F39] The absence of additional forms for the $s\bar{q} \to d\bar{q}$ interaction is evident from the derivation of the minimal operator basis (with dimension $\leqslant 6$ so as not to incur additional $1/m_W^2$ factors) for such processes in Ref. [37]. (An explicit diagrammatic verification is given in Ref. [44]). Note that formal operator product expansion analyses indicate mixing of gauge invariant $s\bar{q} \to d\bar{q}$ operators "into" additional gauge noninvariant operators (explicit calculations require gauge noninvariant counterterms): these operators give no contribution because their matrix elements between on-shell physical (gauge invariant) states vanish [45]. The removal of some possible operators contributing to $s\bar{q} \to d\bar{q}$ requires application of equations of motion (e.g., $Dq = m_q q$). These equations are valid to all orders in perturbation theory: however, with a modified vacuum state (e.g., an instanton), they are altered.

[F40] If $m_{\bar{q}} \neq 0$, one might expect that (as in QED) all infrared divergences should cancel. The calculation of Ref. [46] on a process similar to the one considered here suggests that this cancellation may not occur in QCD.

[F41] Such weak decays are only significant if at least the lowest-lying baryon containing some heavy quark Q is below threshold for a strong decay into a meson containing Q and a light baryon. This condition is well satisfied for Λ and $\Lambda_c(2.1)$. Simple pictures suggest that the effective constituent masses of u, d will enforce this condition for any flavor Q.

[F42] The wavefunction at the origin (squared) for a three-body system bound by harmonic forces differs from that for a two body system bound by the same forces by only a factor 3/4.

[F43] Early reports of $F^* \to F\gamma$ from DASP were not substantiated by Mark II results, while recent Crystal Ball results also fail to exhibit the increase in inclusive η production at $\sqrt{s} \gtrsim 4$ GeV expected from $F\bar{F}$ production and decay.

[F44] Figures 4(a, b) yield decays $D \to s\bar{q}q\bar{q}, s\bar{q}$ $(q = u, d)$, rather than $D \to s\bar{q}s\bar{s}$. A recent report [35] of $D^0 \to K^0\phi$ with a branching ratio of a few percent (comparable to that of $D \to K\pi$) is thus surprising. (Strong decays of low-lying meson resonances very rarely produce $K\bar{K}$ or ϕ except from valence $s, \bar{s}$ in the initial state).

[F45] The two candidate F decays in the emulsion experiment [51] were $F^- \to \pi^+\pi^-\pi^-\pi^0$ and $F^+ \to K^+\pi^+\pi^-K^0$. Hadronic F production is reported with the decay modes $F \to \eta 3\pi$ and $\eta 5\pi$ [54], but $K\bar{K}X$ or ϕX final states are probably suppressed for experimental reasons. There is a very slight indication of $F \to \phi K^0$ in e^+e^- annihilation [53]; if, in fact, such modes dominated the lack of an increase in η production could be accounted for.

[F46] All mechanisms in Fig. 4 contribute equally to these decays (in the relevant local limit $m_W \to \infty$): the process $d\bar{s} \to GG$ through Fig. 4(c) (which could contribute for K^0 but not K^+) is probably unimportant.

[F47] For decay to n massless particles distributed uniformly in available phase space, $\Gamma_n = 16\pi^2/[(4\pi)^{2n}(n-2)!(n-1)!]\Gamma_0$ (the mass parameters appearing in matrix elements for different n are taken to be simply the total decaying particle mass). (PCAC estimates suggest that the relevant mass parameters should instead be $\sim m_\pi$).

[F48] Time reversal invariance (valid to $0(10^{-3})$) requires all weak amplitudes for $K \to \pi\pi$ to be relatively real. Imaginary parts may, however, be introduced by strong reinteractions between the outgoing on-shell pions. The amplitude $\sim e^{i\delta}$ for elastic $\pi\pi$ scattering attains its imaginary component via unitarity through on-shell propagation of π between successive strong interactions. Such final state interactions determine the relative phase of the $I = 0$ and $I = 2$ amplitude for $K \to \pi\pi$ (e.g., [59]). Their effect on the absolute magnitudes of these amplitudes is not calculable.

[F49] An example of this effect is the Gamow/Sommerfeld factor $(\pi\alpha/v)/[1-\exp(-\pi\alpha/v)]$ which accounts for Coulomb interactions between outgoing charged particles (with relative velocity v). Reinteractions between on-shell final particles can give only phase factors (by unitarity): in the Coulomb case the relevant phase shift is divergent by virtue of the infinite range of the Coulomb interactions.

[F50] The original suggestion of such an effect was made in Ref. [61] in the context of the operator product expansion. The necessary anomalous dimensions were then computed in Ref. [62].

[F51] In the physically-irrelevant case of Bose quarks with no color degree of freedom, the symmetry of the total ud wavefunction allows only a $|I_f| = 0$ final state, and implies zero $|\Delta I| = 3/2$ amplitude [63].

[F52] If the initial qq are externally constrained to lie in a color $\bar{3}$ rather than 6 state, then $|\Delta I| = 3/2$ $su \to ud$ processes are forbidden. This is perhaps the case, as discussed below, when the initial su come from a single baryon.

[F53] These results may easily be derived using the direct methods of Ref. [64]. Alternatively, one may write the color part of the amplitude as $T^a_{ij}T^a_{kl}$, where the T^a_{ij} are representative matrices for $SU(N)_c$. Then the SU(N) Fierz identity gives [65] $T^a_{ij}T^a_{kl} = 1/2[\delta_{jk}\delta_{il} - 1/N\delta_{ij}\delta_{kl}]$: taking the parts of this product symmetric and antisymmetric under $i \leftrightarrow k$ yields the required result.

[F54] In a complete treatment of, say, K^0 decay, one must also account for processes such as $s\bar{d} \to GG \to u\bar{u}$ and $u\bar{u} \to GG \to u\bar{u}$, although these give no qualitative modification to the results. In an operator product expansion analysis, they correspond to "mixing" of operators under renormalization. Through such effects the "sea quark" content of the initial meson is accounted for.

[F55] Such a calculation is apparently underway [67].

[F56] To obtain an $SU(3)_c$ *1* qqq system, each qq pair must transform as a $\bar{3}$.

Received October 1980

REFERENCES

1. E. G. Floratos, S. Narison, E. de Rafael: *Nucl. Phys.*, **B155**, 115 (1979).
2. S. Wolfram: "Parton and Hadron Production in e^+e^- Annihilation", talk given at XV Rencontre

de Moriond (March 1980) and Caltech preprint CALT-68-778 (April 1980); R. D. Field, S. Wolfram: "QCD Models for Hadron Production in e^+e^- Annihilation", Caltech preprints in preparation.

3. F. J. Gilman, D. H. Miller: *Phys. Rev.*, **D17**, 1846 (1978).
4. M. L. Perl: "The Tau Lepton", SLAC preprint SLAC-PUB-2446 (December 1979).
5. J. F. Donoghue, K. Johnson: *Phys. Rev.*, **D21**, 1975 (1980).
6. R. Koniuk, R. Leroux, N. Isgur: *Phys. Rev.*, **D17**, 2915 (1978).
7. J. -L. Basdevant, E. L. Berger: *Phys. Rev. Lett.*, **40**, 994 (1978).
8. A. Halprin, B. W. Lee, P. Sorba: *Phys. Rev.*, **D14**, 2343 (1976); C. A. Dominguez: *Phys. Rev.*, **D20**, 802 (1979); R. Holstein, S. B. Treiman: *Phys. Rev.*, **D13**, 3059 (1976).
9. C. Quigg, J. L. Rosner: *Phys. Rep.*, **56C**, 167 (1979).
10. R. Van Royen, V. F. Weisskopf: *Nuov. Cim.*, **1A**, 617 (1967).
11. C. H. Llewellyn Smith: *Phys. Lett.*, **28B**, 335 (1968); *Ann. Phys. (N.Y.)*, **53**, 521 (1969).
12. V. Barger *et al.*: *Phys. Rev. Lett.*, **45**, 83 (1980).
13. V. A. Novikov *et al.* : *Phys. Rep.*, **41C**, 1 (1978).
14. T. J. Goldman, W. J. Wilson: *Phys. Rev.*, **D15**, 709 (1977).
15. A. Stetz *et al.*: *Nucl. Phys.*, **B138**, 285 (1978).
16. K. S. Heard *et al.*: *Phys. Lett.*, **55B**, 324 (1975).
17. J. Bernabeu, R. Tarrach, F. J. Yndurain: *Phys. Lett.*, **79B**, 464 (1978).
18. M. Gronau, C. Llewellyn Smith, T. Walsh, S. Wolfram, T. Yang: *Nucl. Phys.*, **B123**, 47 (1977).
19. M. Suzuki: *Nucl. Phys.*, **B145**, 420 (1979); N. Cabibbo, G. Corbo, L. Maiani: *Nucl. Phys.*, **B155**, 93 (1979); N. Cabibbo, L. Maiani: *Phys. Lett.*, **79B**, 109 (1978).
20. S. Wolfram: "Jet Development in Leading Log QCD", Caltech preprint CALT-68-740 (September 1979). For more details of methods used, see e.g., 21).
21. G. C. Fox, S. Wolfram: "A Gallimaufry of Event Shapes in e^+e^- Annihilation", Caltech preprint CALT-68-723 (July 1979).
22. D. A. Ross: *Nuovo Cim.*, **10A**, 475 (1972).
23. S. B. Libby, G. Sterman: *Phys. Rev.*, **D19**, 2468 (1979).
24. R. E. Marshak, Riazuddin, C. P. Ryan: "Theory of Weak Interactions", Wiley 1969; p. 221; p. 366.
25. R. H. Schindler: "Charmed Meson Production and Decay Properties at the $\Psi(3770)$", SLAC report No. 219 (May 1979).
26. M. Bander, D. Silverman, A. Soni: *Phys. Rev. Lett.*, **44**, 7 (1980).
27. J. Sapirstein: *Phys. Rev.*, **D20**, 3246 (1979).
28. J. D. Bjorken: *private communication*; L. Romans: "A Longer Note on t Quark Detection", Caltech internal memo, *unpublished*.
29. H. D. Politzer, S. Wolfram: *Phys. Lett.*, **82B**, 242 (1979); **83B**, 421E (1979).
30. G. J. Tarnapolsky: *Phys. Lett.*, **79B**, 451 (1978).
31. G. C. Fox, S. Wolfram: *Phys. Rev. Lett.*, **41**, 1581 (1978); *Nucl. Phys.*, **B149**, 413 (1979).
32. B. Guberina, R. D. Peccei, R. Rückl: *Phys. Lett.*, **91B**, 116 (1980); "Dimensional Regularization Techniques and their Uses in Calculating Infrared Safe Weak Decay Processes", MPI preprint MPI-PAE/PTh 56/79 (December 1979).
33. D. A. Ross, C. T. Sachrajda: *Nucl. Phys.*, **B149**, 497 (1979).
34. B. Guberina, S. Nussinov, R. D. Peccei, R. Rückl: *Phys. Lett.*, **89B**, 111 (1979).
35. J. Ellis: *abomination.*
36. M. A. Shifman, A. I. Vainshtein, V. I. Zakharov: ITEP preprint 63–1976 (1976); *Sov. Phys. JETP*, **45**, 670 (1977); *Nucl. Phys.*, **B120**, 316 (1977); *Zh. Ek. I. Teor. Fiz.*, **72**, 1275 (1977).
37. R. K. Ellis: *Nucl. Phys.*, **B106**, 239 (1976); M. A. Shifman, A. I. Vainshtein, V. I. Zakharov: *Phys. Rev.*, **D18**, 2583 (1978); F. Wilczek, A. Zee: *Phys. Rev.*, **D15**, 2660 (1977).
38. N. Vasanti: *Phys. Rev.*, **D13**, 1889 (1976).
39. C. T. Hill: "Higgs Scalars and the Nonleptonic Weak Interactions", Caltech thesis (1977).
40. C. T. Hill, G. G. Ross: "Full Operator Structure of the Nonleptonic $|\Delta s| = 1$ Weak Hamiltonian", Fermilab preprint 79/82-THY (December 1979).

41. J. FINJORD: "The Fate of the Spectator Gluon States, and the Survival of the Penguins", Bern preprint (March 1980).
42. A. I. VAINSHTEIN, V. I. ZAKHAROV, M. A. SHIFMAN: "On the Nonleptonic Decays of K-mesons and Hyperons (Calculation of the Matrix Elements)", ITEP preprint ITEP-64 (1976).
43. C. T. HILL, G. G. ROSS: "Penguins in $\Delta S = 1$ Nonleptonic Weak Decays", Oxford preprint 75/79 (1979).
44. M. B. WISE, E. WITTEN: "A Diagrammatic Analysis of Some Contributions to the $\Delta I = 1/2$ Rule", SLAC preprint SLAC-PUB 2822 (March 1979).
45. C. T. HILL: *Nucl. Phys.*, **B156**, 417 (1979).
46. R. DORIA, J. FRENKEL, J. C. TAYLOR: *Nucl. Phys. B.*
47. C. T. HILL, G. G. ROSS: private communication (June 1980).
48. B. GITTELMAN, P. SKUBIC: "First Physics Results from the CLEO Detector at CESR", Cornell/Rutgers preprint CBX 80–16 (and talk given at 1980 Rencontre de Moriond).
49. A. ALI *et al.*: *Phys. Lett.*, **83B**, 375 (1979).
50. R. SHROCK, L. WANG: *Phys. Rev. Lett.*, **41**, 1692 (1978).
51. J. D. PRENTICE: (FNAL E531), in Proc. 1979 Lepton-Photon Symposium (ed., T. Kirk and H. Abarbanel, Fermilab), p. 563.
52. D. D. REEDER: in PROC. 1979 Lepton-Photon Symposium (ed., T. Kirk and H. Abarbanel, Fermilab), p. 553; L. Montanet; L. Foa, talks at 1980 Rencontre de Moriond.
53. V. LUTH: in Proc. 1979 Lepton-Photon Symposium (ed. T. Kirk and H. Abarbanel, Fermilab), p. 78.
54. F. RICHARD (CERN WA-4), in Proc. 1979 Lepton-Photon Symposium, p. 469.
55. C. BROMBERG *et al.*: "Observations of the D and E Mesons and Possible Three Kaon Enhancements in $\pi^- p \to K^0 K^\pm \pi^\mp \tilde{X}, K^0 K^+ K^- \tilde{X}$ at 50 and 100 GeV/c", Caltech preprint CALT-68-747 (1980).
56. L. B. OKUN: "Weak Interaction of Elementary Particles", Pergamon 1965, p. 150.
57. B. R. HOLSTEIN: *Phys. Rev.*, **D20**, 1187 (1979); N. ISGUR: private communication.
58. B. R. MARTIN, D. MORGAN, G. SHAW: "Pion-pion Interactions in Particle Physics", Academic Press (1976), p. 155.
59. J. GILLESPIE: "Final-state Interactions", Holden-Day (1964).
60. Particle Data Group: *Rev. Mod. Phys.*, **52**, S1 (1980).
61. K. G. WILSON: *Phys. Rev.*, **179**, 1499 (1969).
62. M. K. GAILLARD, B. W. LEE: *Phys. Rev. Lett.*, **33**, 108 (1974); G. ALTARELLI, L. MAIANI: *Phys. Lett.*, **52B**, 351 (1974).
63. C. H. LLEWELLYN SMITH: *Phys. Rev.*, **D1**, 3194 (1970); *Ann. Phys.* (*N.Y.*), **53**, 549 (1969); R. P. FEYNMAN, M. KISLINGER, F. RAVNDAL: *Phys. Rev.*, **D3**, 2706 (1971).
64. R. P. FEYNMAN: "Gauge Theories", Les Houches lectures (1976).
65. P. CVITANOVIC: *Phys. Rev.*, **D14**, 1536 (1976).
66. S. T. BRODSKY: "Perturbative QCD", SLAC preprint 244 (December 1979); M. K. Chase, "QCD Corrections in Doubly Deep Two-photon Processes", Cambridge preprint DAMTP-79/14 (1979).
67. N. DOMBEY, A. KENNEDY: *private communications.*
68. C. SCHMID: *Phys. Lett.*, **66B**, 353 (1977).
69. J. C. PATI, C. H. WOO: *Phys. Rev.*, **D3**, 2920 (1971).
70. S. KAPTANOGLU: *Phys. Rev.*, **D18**, 1554 (1978); H. J. LIPKIN: "Why Most Flavor Dependence Predictions for Nonleptonic Charm Decays are Wrong", Fermilab preprint 79/84/THY (December 1979).
71. S. P. ROSEN: *Phys. Rev.*, **D18**, 2514 (1978).
72. M. B. EINHORN, C. QUIGG: *Phys. Rev.*, **D12**, 2015 (1975).
73. E. FISCHBACH, B. KAYSER, S. WOLFRAM: "Weak Effects in Quarkonium Decays", in preparation (since 1977); E. Fischbach, in "Neutrinos-1978", ed. E. C. Fowler (Purdue, 1978); B. Kayser, "Testing Strong and Weak Gauge Theories in Quarkonium Decays", talk given at Orbis Scientiae 1980 and NSF preprint.
74. F. J. GILMAN, M. B. WISE: *Phys. Rev.*, **D19**, 976 (1979).
75. F. E. CLOSE, H. R. RUBINSTEIN: "Short and Long Distance Effects in Weak Radiative Decays", Rutherford preprint RL-80-013 (March 1980).

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СЛАБЫЕ РАСПАДЫ

Резюме

Слабые распады странных, шармовых и более тяжелых мезонов обсуждаются в рамках квантовой хромодинамики.

S. Wolfram

ROZPADY SŁABE

Streszczenie

W ramach chromodynamiki kwantowej przedyskutowano rozpady mezonów dziwnych, powabnych i mezonów cięższych.